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An Important Inconsistency at the Heart of the Standard Macroeconomic Model

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I. The Problem Stated

The standard neoclassical model is the foundation of most mainstream macroeconomics. Its basic structure dominates the analyis of macroeconomic phenomena, the teaching of the subject, and even the formation of economic policy. And of course the modern quantity theory of money and its attendant monetarist prescriptions are grounded in the model's strict separation between real and nominal variables.

It is quite curious, therefore, to discover that this model contains an inconsistency in its treatment of the distribution of income. And when this seemingly small discrepancy is corrected, without any change in all of the other assumptions, many of the model's characteristic results disappear. Two instances are of particular interest. First, the strict dichotomy between real variables and nominal variables breaks down, so that, for example, an increase in the exogenously given money supply changes real variables such as household income, consumption, investment, the interest rate, and hence real money demand. Secondly, since the price level depends on the interaction of real money demand and the nominal money supply, and since the former is now affected by the latter, price changes are no longer proportional to changes in the money supply. Indeed, we will demonstrate that prices can even *fall* when the money supply rises. The link to the quantity theory of money, and to monetarism, is severed.

In its most basic form, the model encompasses four "markets": commodities, labor, private bonds, and money[1]. These arenas are bound together by the (implicit) household and business sectors' budget constraints, which link what agents plan to spend with what they expect to receive. When cast in Walrasian terms, these budget constraints aggregate into the familiar expression known as Walras' Law, which states that the sum of the planned demands for the four items must equal the sum of their expected supplies -- i.e. that excess demands in the four arenas must sum to zero (Clower 1979, Buiter1980). This latter result is then used to justify the dropping of any one market from the formal description of the model, on the grounds that equilibria (or even particular disequilibria) in any three determine the state of the fourth. In the standard form depicted in equations 1-11 of the next section, it is the bond market which drops out of view (McCafferty 1990: 46).

As is well known, the standard model exhibits a block recursive structure beginning from equilibrium in the labor market and moving to real output, demand and its components including the real demand for money, and ending finally in nominal wages and prices. The price level in particular is determined by the conjunction of the real demand for money and a given nominal money supply. Since the former is a function of real variables such as output and the interest rate, and since the block recursive structure implies that real variables are unaffected by the money supply (because they are analystically upstream of nominal relations), it follows that doubling the money supply must double prices so as to keep the real money supply equal to an unchanged real money demand. This is acknowledged to be an absolutely central result of the model (McCafferty 1990, p. 53). Yet it turns out to be very fragile indeed.

The source of the problem lies in the apparently innocous assumption that all of the real net income of the business sector (the real value of the net product) is somehow distributed to households. In the case of wage income, this is straightforward, since firms pay workers for their labor services. But when we ask how profits are to be distributed, we find that within the logic of the model they can only be distributed in the form of interest payments on the bonds issued by firms, *for there is no other instrument available in the model.* Firms borrow money from households by issuing bonds, and are then obliged to pay interest on them at the rate determined by the model. The difficulty is that these aggregate real interest payments will generally differ from aggregate real profits. This in turn implies that household income (wage and interest income) must generally differ from business income (wages and profits).

It is a simple matter to correct the model by explicitly writing real household income as the sum of real wage and interest income (the latter being the interest rate times the real value of bonds). On the side of businesses, this implies that the value of new bonds issued by firms (their new borrowing) in a given period can differ from the value of the investment expenditures they plan to make, precisely because their total outpayments to households can differ from their own net income. Budget constraints, after all, only require that the overall sum of inflows equal overall outflows. With these minor changes, the model becomes consistent.

But while the correction appears minor, its consequences are not. The full employment core of the original model is preserved, so that real wages, employment and output continue to be the same. This means that real profits are also unaffected. But now a change in the price level (due, say, to a change in the money supply) changes the real value of bonds outstanding, and hence changes the level of real interest flows [2]. Since real interest flows enter into household income, this affects real consumption demand, real investment demand (which is the difference between the unchanged real output and changed consumption demand), and the interest rate (which must adjust to make real investment demand come out right). Because real money demand is affected in opposite ways by real household income[3] and the interest rate, both of which change in the same direction, its overall direction of change is ambiguous. It can rise or fall in the face of an increase in the money supply so that prices can change less or more than the money supply. This property alone is sufficient to sever any simple linkage between the two. As noted earlier, we can show that even under perfectly plausible parameter values, prices can actually fall when the money supply increases.

The problem that we have identified is noted in passing in Patinkin's (1965) seminal text, but is then buried in footnotes. In an effort to maintain a forced equality between aggregate household income and aggregate value added, he is driven to make a series of *ad hoc* behavioral assumptions. He does not remark on the contradictions to which these give rise. We comment on his proposed solutions in section II.3.

One implication of our results is that the bond market can no longer be "dropped" out of the story. This is because real interest payments depend on the number of bonds, which requires us to deal explicitly with the determinants of this quantity. It is true, of course, that Walras's Law still allows us to infer the state of excess demand in the bond market from that in the other three arenas. But this implicit relation between the supply and demand for bonds does not in itself allow us to determine their respective levels. For that, and hence for the determination of real interest flows, the bond market becomes a structurally necessary part of the model. This is possible because a description of the bond market actually requires *two* conditions: Walras' Law, which in this model reduces to the requirement that the bond market be in equilibrium; and an investment finance constraint for the firm, which provides us necessary additional equation. We will see that these two conditions derive from the implicit budget constraints of the household and business sectors (Buiter 1980).

II. A Formal Exposition

1. The Standard Neoclassical Macroeconomic Model

We start with the standard exposition of the model, elaborated to as to make explicit its underlying assumption that household income is identical to value added -- i.e. that profits are always completely distributed. Thus we explicitly express consumption and money demand functions in terms of household income (equations 4 and 6), and then add the condition that household income equal value added (equation 11). This has no effect on the results at this stage in the argument, but it does prepare us for what follows. In general, lower case refers to real, and upper case to nominal, variables.

Theory of the firm

1. ys = f(k, nd) [aggregate production function, with given real capital stock k]
2. nd = nd(W/P) [P = MC, where MC=W/MPL, MPL = f(nd) from short run profit-maximizing]
3. id = id(r) [id(r)= invest demand]

Theory of the household

4. cd = cd(yh) [consumption function, from utility-max. behavior]
5. ns = ns(W/P) [labor supply of households, from utility-max. behavior]
6. Md/P = md(yh, r) [money demand function of households, from optimal portfolio formation]

Definitions and equilibrium conditions

7. yd = cd + id [definition of aggregate demand]
8. yd = ys [commodity market equilibrium]
9. nd = ns [labor market equilibrium]
10. Md = M [money market equilibrium, the money stock M being taken as given]

Distribution condition

11. yh = ys [household income assumed to equal value added, i.e. all profits are distributed]

where, respectively,
yd, ys = real commodity demand and supply
nd,ns = labor demand and supply
yh = real household income
cd, id = real consumption and investment demand
Md = nominal money demand
r = the real (and nominal) interest rate
W,P = nominal wages and profits
M = the exogenously given money supply

Note that we have 11 endogenous variables defined above (M being exogenous), and 11 independent equations.

A fundamental characteristic of the model is that it is block recursive. Thus equations 2,5,9 determine the equilibrium real wage (W/P)* and real employment n*, and through equations 1, 8 the latter determines real output and real demand y*. The preceding variables then determine equilibrium household income yh*, consumption c*, investment i*, the interest rate r*, and real money demand (Md/P)* = md*= md(y*, r*), by means of equations 3,4,6,7,11. This last variable, in conjunction with the given money supply M and equations 6,10 allows us to determine nominal money demand Md = M, the nominal price level P = Md/md(yh*, r*), and the nominal wage W = P (W/P)*. The significance of block recursion is that downstream variables have no effect on upstream ones. Therefore a change in the supply of money M must change P in the same proportion and direction, because P = M/md*, and the equilibrium real output y* and interest rate r* which determine real money demand md* are upstream of P (and independent of M). It is this particular property which is the foundation for the monetarist aspect of the model. And it is precisely this property which does not survive.

2. Finding the Bond Market

Although interest *rates* play an important role in the operations of the model, there is no representation of interest *payments* . Where the subject is mentioned at all, it is generally dismissed on the grounds that Walras' Law allows us to drop the bond market out of explicit consideration (Patinkin 1954, p. 125; Modliagni 1963, p. 81; Patinkin 1965, p.230; Barro 1990, p.108; McCafferty 1990, p. 46). But Walras' Law only permits us to deduce that there will be equilibrium in the bond market if the other three are in equilibrium. It does not tell us what the equilibrium quantity of bonds, and hence what the equilibrium level of interest payments, will be.

Most importantly, it does not permit us to drop the flow of interest payments out of sight.

The issues involved can be brought into focus by considering the *ex ante* budget constraints which underlie the whole model, because then we are forced to explicitly account for the planned uses and expected sources of funds (including borrowing) for each sector. In Table 1, each column represents a particular sector's uses (negative signs) and sources (positive signs). If sectors' are consistent in making their plans[4], each column, and hence the overall sum of columns, must sum to zero.

The row sums of the matrix are another matter, since they represent the discrepancy between *ex ante* expenditures planned on a particular activity by a given sector and the *ex ante* receipts expected from the same activity by another sector. There is no reason here for individual rows to sum to zero, since plans by one sector need not match anticipated receipts by another. All that is required is that the overall sum of the rows be zero[5], since this is merely the overall column sum. The latter requirement implies that *ex ante* discrepancies must add up to zero, which in this context is simply Walras' Law.

In Table 1, flows are presented in real terms, and the initial number of bonds is denoted by b0 (so that bd - b0 represents the change in bond holdings desired by households, and bs - b0 represents the change in bond issue expected by firms). Of crucial significance are the *yet undefined* flows of real financial payments f^e expected by households and f^p planned by firms.

Table 1: The *Ex Ante* Flow of Real Funds

	Households	Firms		**Totals**
Consumption and investment	- cd	- id		- yd = - (cd + id)
Sales		ys		ys
Wages	(W/P)ns	- (W/P)nd		- (W/P)·(nd - ns)
Financial payments	f^e	$- f^p$		$(f^e - f^p)$
Changes in bonds	- (Pb/P)·(bd - b0)	(Pb/P)·(bs - b0)		- (Pb/P)·(bd - bs)
Changes in money	- (Md - M)/P			- (Md - M)/P
Totals	0	0		0

The flow of funds matrix implies that there are two additional equations implicit in the model. We can derive these equations by setting the sum of elements of any two columns equal to zero (since any two imply the third). Taking the firms' and totals columns give us the most familiar results.

Thus if we take the column sum for firms', recognizing that ys - (W/P)·nd = real profits = Pi, and that Pi - f^p = undistributed profits, we find that the sectoral budget constraint of firms is equivalent to an *investment finance constraint* which says that the real value of new bonds issued must equal the *excess* of investment needs over undistributed profits.

12. $(Pb/P)\cdot(bs - b0) = id - [ys - (W/P)\cdot nd] - f^p = id - (Pi - f^p)$ [investment finance constraint]

For the other equation we take the total column sum (and reverse signs), which gives us an expression recognizable as Walras' Law (equation 13), except for the presence of the yet undefined financial payments flows. Indeed equation 13 *is* exactly the form of Walras' Law which Buiter (1980) derives[6]. We will return to that point shortly.

13. $(yd - ys) + (W/P)\cdot(nd - ns) + (Md - M)/P + (Pb/P)\cdot(bd - bs) - (f^e - f^p) = 0$ [Walras' Law]

Real financial payments appear in both of the preceding relations. But what determines them? The answer lies in the fact that the model assumes that firms issue new bonds, in which case they must also pay interest on these same bonds. *Since bonds are the only instruments for the disbursement of profits, these interest flows are the only financial payments dictated by the logic of the model*. If, in a Walrasian spirit we assume that borrowing is planned at the beginning of the period and that the corresponding interest rate flows are expected during that same period, and if we note that the price of bonds Pb = 1/r, then[7]

14. f^e = interest payments expected by household = r·(Pb/P)·bd = bd/P = real value of bonds demanded

f^p = interest payments planned by firms = r·(Pb/P)·bs = bs/P = real value of bonds supplied

Substituting the expressions for real financial payments (equation 14) into Walras's Law (equation 13) allows us to combine the resulting bond market terms into one expression concerning excess demand in the bond market: (Pb'/P)·(bd - bs), where Pb' = Pb(1 - r) = the *net* price of bonds. Note that the three equilibrium conditions in equations 8-10, along with Walras' Law in equation 13 imply the bond market equilibrium condition bd = bs. With this elaboration, the model is completely specified.

The trouble is that now the overall model, built around the familiar core in equations 1-11 from which all the standard results derive, *is inconsistent*. This is because the standard form *assumes* that household income yh = the value of net output y = wages + profits. But in actuality yh = wages + interest payments = (W/P)·ns + r·(Pb/P)·bd = (W/P)·ns + bs/P,*and the two are not equivalent because real interest payments will not generally equal real profits*. The former is determined in the bond and money markets, and the latter is dermined by a given capital stock and the full-employment marginal product of capital. They would be equal only by accident.

Removing the inconsistency is straightforward. One only has to substitute the proper expression for yh into what was formerly equation 11 of the original model.The consistent model then consists of equations 1-10 previously, this corrected definition of household income (equation 11'), equations 12 and 13 modified to reflect the definitions of financial payments in equation 14 into account, and an explicit definition of bond price Pb:

11'. yh = wages + interest payments = (W/P)·ns + bs/P [household income]

12'. (Pb/P)·(bs - b0) = id - ([ys - (W/P)·nd] - r·(Pb/P)·bs) [investment finance constraint]

13'. (yd - ys) + (W/P)·(nd - ns) + (Md - M)/P + (Pb'/P)·(bd - bs) [Walras' Law]

where Pb' = Pb·(1 - r) = net price of bonds.

14'. Pb = 1/r

Now the model is consistent. But its behavior is substantially different. This is because household income depends on the real value of interest payments, which means that a rise the money supply affects both the price level *and* the level of real income (throught the real value of interest flows, in equation 11'). This in turn raises real consumption and *cet. paribus*, also raises real money demand (equations 4,6), both of which depend positively on real household income. Because real output, and hence aggregate demand (equation 8) is unaffected, the fact that consumption demand rises implies that real investment demand must fall and hence the interest rate must rise. *Therefore a rise in the money supply can raise the interest rate and "crowd out" investment.*

Real household income and the interest rate move together but have opposite effects on real money demand (equation 6), so the overall effect ambiguous. But the important point is that real money demand md(yh, r) generally change when the money supply changes. Since the price level P = M/md(yh, r) This means that neither the magnitude, nor even the direction, of price changes is a simple reflection of changes in the money supply. As illustrated in Table 1 and Appendix A, analysis and simulations reveal that some real effects can be substantial, and that prices can even *fall* when the money supply increases.

Table 1: Simulated price and real variable changes in the face of an increase in money supply

M	y	yh	b	c	i	r	W	P
3.8	0.981	0.981	1.087	0.589	0.393	0.172	3.934	2.768
4.2	0.981	0.965	0.943	0.579	0.402	0.044	3.558	2.504
(10.5%)	(0%)	(-1.6%)	(-13.2%)	(-1.7%)	(2.3%)	(-25.56%)	(-9.6%))	(-9.5%)

3. Patinkin's comments on the issue

The crux of the problem which we have identified lies in the fact that within the logic of the neoclassical model, profits and real interest payments are differently determined and hence will not be generally equal. Making these flows distinct render the model consistent. But then its standard results, particularly those pertaining to the so-called dichotomy between real and nominal variables, and to the putative effects of a change in the money supply, no longer hold.

Conversely, the standard results require that real business financial outpayments f^p = real profits mpkk at all times, where financial payments at least encompass real interest flows rPbbs/P. Only then will household income yh = net value added y, and the value of new bonds issued bonds equals the value of investment (from equation 12). Since all the relevant variables are either given exogenously or determined within the model, *one must introduce a new variable to bring about the desired result.* We will see this is precisely what Patinkin attempts to do.

Throughout his text, Patinkin (1965) *assumes* that all profits will be automatically distributed. But the problems we have raised also seem to have troubled him, because he does make an attempt, albeit very cursory, to justify this crucial assumption. There, he notes that his analysis makes it necessary to assume that any excess of profits over interest payments is "appropriated by entrepeneurs" (Patinkin 1965, 201), so that by construction total financial outpayments by firms f^p = real profits mpkk. Nowhere does he even mention the fact that the difference between profits and interest payments can be positive or negative, which would require entrepeneurs to always pay themselves bonuses in the first case, and always assess themselves *penalties* in the second. Moreover, he does not note that if they did hapen to behave in such a manner, the excess profits they paid themselves would be taken from funds which would otherwise be used for investment, and which then have to be made up by extra borrowing by their firms. They would be simply robbing Peter to pay Paul. The implicit behavioral assumptions become even more strained when one considers the opposite case in which interest flows exceed profits, for then entrepeneurs must be supposed to reduce their wage incomes (via a penalty) so as to make up the difference. In all cases, there is absolutely no motivation within the model's own microfoundations for any such behavior -- which given Patinkin's emphasis (and that of neoclassical macroeconomics in general) on the importance of microfoundations, is very telling indeed.

One implication of the assumed automatic full disbursement of profits is that firms must finance investment entirely through borrowing in the bond market (equation 12). This in turn implies that in both real and nominal terms the total value of bonds equals the value of the stock of capital. Just a few pages later, Patinkin runs headlong into the problems caused by this assumption. And once again, he is forced to make *ad hoc* assumptions in order to keep these new difficulties at bay.

In the course of a discussion of the effects of a doubling of the money supply, he derives the familiar result that whereas the price level is P doubled, real variables such as output y, the interest r (and hence bond price Pb = 1/r), and the real money supply M/P are unchanged. The real value of the planned bond supply Pbbs/P is presumed to be a function of these real variables, so it too is unchanged. But in that case the *number* of bonds issued by firms bs must somehow be doubled as nominal variables double (Patinkin, 1965, pp. 216-17). So, in a footnote, he says:

> There is an implicit assumption here that all the firms' capital equipment must be replaced during the period in question (*ibid* , p.217, footnote 13).

But what can it mean that the firms capital equipment must be "replaced", and how could this resolve the present difficulty? The answer lies in recognizing that with y and r are unchanged and P doubled, real net investment is unchanged and nominal investment is doubled. Thus firms will have to issue a new quantity of bonds equal to the changed nominal value of new investment. However, with the price level doubled, the nominal value of new capital will also have doubled, so *if firms are to maintain a stock of bonds equal to the value of the capital stock, as required by the distributional assumption* , they must sell a quantity of new bonds equal to the changed nominal value of the capital stock. These two distinct requirements are only consistent as long as real investment and the real capital stock are always equal -- i.e. only if *all capital turns over in one period* [8]. Here, firms are assumed to issue bonds to finance new investment, so that $Pb \cdot \Delta bs = \Delta Pi$, and since $i = \Delta k = k$, we also have $Pb \cdot \Delta bs = \Delta P \cdot k$.

Understandably uneasy about the previous solution, Patinkin proposes yet another one.

> Alternatively, we can assume that firms immediately write up their capital equipment in accordance with its increased market value, sell additional bonds to the extent of this increased value, and pass on the explicit capital gains to their respective entrepeneurs. Conversely, in the event of a decrease in prices, entrepeneurs must make good the implicit capital loss, and firms then use these funds to retire bonds. *In this way the nominal amount of bonds outstanding can always be kept equal to the current value of the firms' assets* (*ibid* , p.217, footnote 13, emphasis added).

Recall that the crux of the problem is that the assumed automatic distribution of profits requires that the nominal value of bonds remain equal to the nominal value of the capital stock. So now Patinkin is assuming that firms no longer issue bonds to finance new investment, but rather issue them to realize capital gains on the existing capital stock: $Pb \cdot \Delta bs = \Delta P \cdot k > \Delta P \cdot i$, since in general $k > i$.

A simple numerical example illustrates the difficulty facing Patinkin. Suppose that initially $Pb = 5$, $P = 1$, $i = 10$, $k = 100$, and that a change in the money supply produces $\Delta P = 1$. Then if new bonds are issued to finance the changed value of new investment, $Pb \cdot \Delta bs = \Delta P \cdot i = 10$, so $bs = 2$. Alternatively, if new bonds are issued to realize capital gains on the stock of capital, $Pb \cdot \Delta bs = \Delta P \cdot k = 100$, so $\Delta bs = 20$. The two solutions are inconsistent unless one assumes that all capital turns over in one period ($k = i$ at all times), or one abandons the notion that firms issue bonds to finance nominal new investment in favor of the assumption that bonds are issued to "pass on the explicit capital gains [from the increased value of the capital stock] toentrepeneurs".

In both cases, these highly strained and completely unmotivated behavioral assumptions are driven entirely by the need to avoid the contradictions generated by the requirement that household income be always the same thing as the aggregate net income of firms. We have already seen that such an equality is not sustainable within the logic of the model. Patinkin's discussion only verifies that fact.

III. Summary and Conclusions

Our central finding has been that the famous dichotomy between real and nominal variables which emerges from the standard neoclassical macroeconomic model rests on extraordinarily shaky foundations. Writing out the *ex ante* flow of funds corresponding to the model reveals that its standard form embodies inconsistent assumptions about the treatment of the distribution of non-wage income. Firms are assumed to pay out all of profits, but the only instrument available is the interest on the bonds they have issued. Contrary to the implicit assumption within the model, the resulting interest flows will not generally equal profits.

The revealed inconsistency is easily rectified by distinguishing between household income (wages and interest payments) and net value added (wages and profits). But then, leaving all other assumptions unchanged, the model's behavior changes dramatically. In particular, real variables such as consumption, investment, the interest rate and real money demand become intrinsically linked to nominal variables such as the price level and the money supply. One striking consequence of this is that a rise in the money supply can actually lead to a fall in prices -- even under the standard assumptions about money demand functions. Monetarism cannot be grounded in a consistent neoclassical model.

The corrected model is perfectly consistent. While we ourselves do not advocate such a model because it is still entirely neoclassical in its construction, it our hope that our colleagues in that tradition will recognize it as a consistent exposition of their own framework.

Appendix A: Numerical Simulation of the Consistent Neoclassical Model

The Corrected Model

1. $ys = a \cdot k^{ß} \cdot nd^{1-ß}$
2. MPL $(1-ß) \cdot nd^{-ß} = W/P$
3. $id = \square_0 - \square_1 \cdot r$
4. $cd = \square \cdot yh$
5. $ns = ó_0 (W/P)^{ó1}$
6. $Md/P = \lambda_0 + \lambda_1 \cdot yh - \lambda_2 \cdot r$
7. $yd = cd + id$
8. $yd = ys$
9. $nd = ns$
10. $Md = M$

11'. $yh = (W/P) \cdot ns + (r \cdot Pb \cdot bd)/P$ [household income]

12'. $(Pb/P) \cdot (bs - b0) = id - (ys - (W/P) \cdot nd - r \cdot Pb \cdot bs/P)$ [investment finance constraint]

13'. $(yd - ys) + (W/P) \cdot (nd - ns) + (Md - M)/P + (Pb'/P) \cdot (bd - bs) = 0$ [Walras' Law]

where $Pb' = Pb \cdot (1 - r)$ = net price of bonds.

14'. $Pb = 1/r$

We have 14 endogenous variables (ys, nd, id, cd, ns, yd, yh, Md, r, W, P, Pb, bs, bd) and 14 independent equations. The three equilibrium conditions and Walras' Law (equation 8-10, 13) togethe imply bond market equilibrium bd = bs.

Parameter values:

a = 0.97	ß = 0.4	k = 3.86	□0 = 0.4054	□1 = 0.75	□= 0.6	
ó0 = 0.4	ó1 = 0.1	λ0 = 0.20	λ1 = 1.65	λ2 = 2.6	b0 = 0.9	

Initial values (note that household income has been *initially* set equal to net value added -- i.e. all profits are initially distributed):

<u>M = 3.8</u>

ys = yd = 0.981 ns = nd = 0.414 Md = M = 3.8 bd = bs = 1.087 cd = 0.589 id = 0.393

yh = 0.981 [note that yh = ys, initially] r = 0.172 Pb = 5.81 W = 3.934 P = 2.768

Now when the money supply rises by 10.5% to M = 4.2, real output and employment are unchanged, household income changes only slightly (from 0.981 to 0.965), and yet there are substantial changes in the interest rate (it drops from 17.2% to 4.4%), and *the price level actually falls by 9.5%* .

<u>M = 4.2 (+10.5%)</u>

ys = yd = 0.981 ns = nd = 0.414 Md = M = 4.2 bd = bs = 0.943 cd = 0.579 id = 0.402

yh = 0.965 r = 0.044 Pb = 22.721 W = 3.558 P = 2.504 (- 9.5%)

Analysis of the consistent model helps us understand how this sort of result can occur. Equibrium in the labor market and the aggregate production function (equations 1,2,5,9) yield equilibrium real output y*, the real wage bill (W/P)*n*, and real profits $\square$* = y* - (W/P)*n* = mpk*k*, none of which are affected by nominal changes. Then equilibrium in the commodity market and its associated relations (equations 3,4,7,8) gives us

$y^* = cd^* + id^* = \cdot yh + \square_0 - \square_1 \cdot r$

A comparable relation can be derived from money market equilibrium and associated conditions. (equations 6,10).

$M/P = \lambda_0 + \lambda_1 \cdot yh - \lambda_2 \cdot r$

Note that the two derived relations do not reduce to the familiar I-S, L-M pair because real household income yh is not generally equal to real (full employment) output y*. The former depends on the real demand for bonds, and it is precisely this dependence that prevents us from "dropping" the bond market out of sight. From equations 8-10 and 13' we get bd = bs = b, so from 12', 14', 7, 4, and 11',

$(1/r)(b/P - b0/P) = id - ys + (W/P)^*n^* + b/P = -cd + yh = (1-\square)yh$

$[yh - (W/P)^*n^* - b0/P] = r\,(1-\square)yh$

$[1 - r\cdot(1-\square)]yh = [b0/P + (W/P)^*n^*]$, where since the propensity to consume $\square < 1$, $yh > 0$ if $r < 1$.

Substituting this last expression into each of the first two gives us two nonlinear equations in 1/P and r[9]

, whose intersection determines the equilibrium values of P*, r*. The value of the money stock M enters directly into the equilibrium values via equation B, as does the initial number of bonds b0 via both equations.

A. $1/P = [\ \{(1 - r + r\cdot\square)(y^* - \square_0 + \square_1\cdot r)/\square\} - (W/P)^*\cdot n^*]/b0$

B. $1/P = [(1 - r + r\cdot\square)(\lambda_0 - \lambda_2\cdot r) + \lambda_1\cdot(W/P)^*\cdot n^*]/[(1 - r + r\cdot\square)\cdot M - \lambda_1\cdot b0]$, for $(1 - r + r\square)\cdot M \square \lambda_1\cdot b0$

Given the particular functional forms used in this appendix, one can impose restrictions on r (e.g. $y^* > \square_0 - \square_1\cdot r > 0$ since the right hand side is id, and $1 - r + r\square > 0$ since that is necessary for yh >0, etc.). There are multiple intersections possible for such nonlinear curves, hence multiple possible equilibria. Plotting these curves and their shifts as M or bo change gives one some sense of the complexity of the possible effects.

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Notes

1. The desired holdings of money are counterposed to and exogenous supply of money, which is not really a market.

2. Real interest payments r·Pb·b/P = b/P = the real value of bonds outstanding, where r = the rate of interest, Pb = the price of bonds = 1/r, b = the number of bonds, and P = the price level.

3. In the standard model, only households hold money. But this is not essential to our results.

4. Clower (1979, p. 297) calls this assumption "a fundamental convention of economic science".

5. Sectoral budget constraints imply that individual columns, and hence both the sum of column sums and the the sum of row sums, equal zero.

6. Buiter (1980, equation 14, p. 6) actually lists the financial payments as "dividend" payments expected and planned. This is odd because the model contains bonds but no equity (were it the other way around there would be no rate of interest in the model). In leaving these "dividend" unexplained, he sidesteps the inconsistency which we have identified.

7. An alternate assumption is that interest flows in a given period are on the stock of bonds inherited from the previous period (b0). In this case, $f^e = f^p$ = r·(Pb/P)·b0 = b0/P = current real value of the opening stock of bonds. Then equation 13 takes the familiar form of Walras' Law, since the term $(f^e - f^p)$ drops out. But the dependence of investment finance on interest payments (equation 12), and hence on undistributed profits, still remains. And so the basic contradiction in the standard model continues to exist.

8. Formally, the number of new bonds issued is given by the investment finance relation Pb(bs - b0) = Pk. In the standard model, with r=1/Pb and i=k unchanged, a change in the money supply implies Pbbs = Pk in this particular period alone. Hence only if capital turns over in one period -- i.e. if there is no fixed capital -- does this also imply that the outstanding stock of bonds will have doubled.

9. The first of these is straightforward, and results in equation A. For the second, we get

M/P = $_0$ + $_1$·yh - $_2$·r = M/P = $_0$ + $_1$· [{b0/P + (W/P)*n*}/ {1 - r (1-)}] - $_2$·r, which after rearrangement yields equation B.