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# A Simplified Stock-Flow Consistent Post-Keynesian Growth Model

by

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In a series of papers (e.g. Zezza and Dos Santos, 2004; Dos Santos, 2004a, 2004b) we have argued that the so-called "stock-flow consistent approach" (SFCA) to macroeconomic modeling not only provides a rigorous foundation for post-Keynesian macroeconomics but is also a relatively unexplored frontier of (various schools of) Keynesian macroeconomics[1].

We have noted also that the increase in analytical rigor allowed by the SFCA does not come without a price. More often than not, SFC models are too big to be analytically treatable and can only be analyzed with the help of computer simulations. Since the "reality" post-Keynesian SFC models try to approximate is complex, this is hardly surprising[2]. On the other hand, we do acknowledge that an analytically treatable version of our models would help the presentation of our ideas considerably. This paper aims precisely to present one such version.

In fact, we believe that the model presented here—which builds on previous efforts by Godley and Cripps (1983), Godley (e.g. 1996, 1999), Lavoie and Godley (2001-2002), Taylor (1991), and Tobin (e.g. 1980, 1982), among others—is intuitive and general enough to be considered a "baseline" (didactic) SFC post-Keynesian model[3]. As we hope to make clear to the reader, it sheds light on a wealth of classic post-Keynesian macroeconomic issues, and (just like the old IS/LM model) can easily be modified to address several other ones (or the previous ones from different theoretical perspectives).

What follows is divided in four parts. First we present the "structural" hypotheses of the model and the logical (accounting) constraints imposed by them. Second, we "close" the accounting constraints with a specific set of post-Keynesian behavioral hypotheses[4]. Third, we discuss the "short period" and "long period" properties of our specific "closure." We finish with a brief discussion of possible extensions and simplifications of the model.

[1] Essentially the same points were noted well before us—with different terminologies—by Tobin (1980, 1982), Godley and Cripps (1983), and Lavoie and Godley (2001-2002), among others.
[2] The adjective post-Keynesian is used here in the sense of Palley (1996) and Lavoie (1992).
[3] Conceived as a simplified version of Zezza and Dos Santos (2004), the model presented here ended up being very close in spirit to "the" "heterodox model" by Foley and Taylor (2004).
[4] Though, following Taylor (1991, 2004), we readily agree that several other "closures" are possible. Moudud (1998), for example, presents a "classical" analysis of an economy similar to the one above.

## 1. THE STRUCTURE OF OUR ARTIFICIAL ECONOMY

The economy assumed here has households, firms (which produce a single good, with price $p$), banks and a government sector[5]. The aggregated assets and liabilities of these institutional sectors are presented in table 1 below.

Table 1 summarizes several theoretical assumptions. First and foremost, the economy assumed here is a "pure credit" one, i.e. all transactions are paid with bank checks[6]. Moreover, the banking sector is supposed to remunerate deposits at the T-bill rate (making profits only through their loans to firms), so households do not care to buy T-bills themselves, keeping their wealth only in the form of bank deposits and equities[7]. The banking sector is also assumed to: (i) accept government debt as means of payment for government deficits[8]; (ii) not pay taxes; and (iii) to distribute all its profits (so its net worth is equal to zero)[9]. Finally, firms are assumed to finance their investment using loans, equity emission and retained profits. The Modigliani-Miller (1958) theorem does not hold in this economy, so the specific way firms choose (or find) to finance themselves matters and their net worth is not necessarily zero (i.e. Tobin's $q$ is not necessarily 1)[10].

[5] Dos Santos (2004b) argues that these are quintessential features of the economies studied by "Financial Keynesian" authors (such as Davidson, 1972; Godley, 1999; Minsky, 1986; and Tobin, 1980). Similar economies have been studied in a long series of papers by Reiner Franke, Willi Semmler, and associates, at least since Franke and Semmler (1989).

[6] A similar simplifying hypothesis is adopted in Godley and Cripps (1983, chapter 5). Section 4 discusses the implications of relaxing it.

[7] According to Stiglitz and Greenwald (2003, p.43), a banking sector with these characteristics "is not too different from what may emerge in the fairly near future in the USA." In any case, this hypothesis allows us to simplify the portfolio choice of households considerably. More detailed treatments (such as the ones in Tobin, 1980; or Lavoie and Godley, 2001-2002) can easily be introduced, though only at the cost of making the algebra considerably heavier (see section 4 for a discussion).

[8] Here we will work with the conventional case in which $B>0$, noting that not too long ago (in the Clinton years, to be precise) many analysts were discussing the consequences of the U.S government paying all its debt. A negative B (i.e. a positive government net worth) would be interpreted in this model as "net central bank advances" to the banking sector as a whole.

[9] We are also simplifying away banks' (and government's) investment in fixed capital, as well as their intermediary consumption (wages, etc). These assumptions are made only to allow for simpler mathematical expressions for household income and aggregate investment.

[10] As Delli Gatti et al. (1994, footnote 13) point out, " the greater the ratio of equity to debt financing the greater the chance that the firm will be a hedge financing unit." This "Minskyan" point is, of course, lost in models in which firms issue only one form of debt.

| Table 1: Aggregate Balance Sheets of the Institutional Sectors. *pe* stands for the price of one equity. | | | | | |
|---|---|---|---|---|---|
| | Households | Firms | Banks | Government | Row Totals |
| 1-Bank Deposits | $+D$ | | $-D$ | | $0$ |
| 2-Bank Loans | | $-L$ | $+L$ | | $0$ |
| 3-T-Bills | | | $+B$ | $-B$ | $0$ |
| 4-Capital Goods | | $+ p \cdot K$ | | | $+ p \cdot K$ |
| 5-Equities | $+E \cdot pe$ | $-E \cdot pe$ | | | $0$ |
| 6-Net Worth (Column Totals) | $+ Vh$ | $+ Vf$ | $0$ | $-B$ | $+ p \cdot K$ |

Table 2 below shows the "current flows" associated with the stocks above. As such, it (rigorously) represents very intuitive phenomena[11]. Households in virtually all capitalist economies receive income in the form of wages, interest on deposits, and distributed profits (of banks and firms) and use it to buy consumption goods, pay taxes and save (as depicted in the households' column of table 2)[12]. The government, in turn, receives money from taxes and uses it to buy goods from firms and pay interest on its (lagged stock of) debt, while firms use sales receipts to pay wages, taxes, interest on their (lagged stock of) loans, and dividends, retaining the rest to help finance investment. Finally, banks receive money from their loans to firms and holdings of Treasury bills and use it to pay interest on households' deposits and dividends. In a "closed system" like ours every money flow has to "come from somewhere and go somewhere" (Godley, 1999, p.394), and this shows up in the fact that all row totals of table 2 are zero.

[11] A numerical example in section 3 aims to provide further assistance to the reader in understanding exactly how the artificial economy presented here operates.
[12] We are simplifying household debt and housing investment, however.

| Table 2: “Current” transactions in our artificial economy<br>A (+) sign before a variable denotes a receipt while a (-) sign denotes a payment. | | | | | | |
|---|---|---|---|---|---|---|
| | Households | Non Financial Firms<br>Currrent | <br>Capital | Govt. | Banks | Row Totals |
| 1-Cons. | $-C$ | $+C$ | - | - | - | $0$ |
| 2-Govt. Expenditures | - | $+G$ | - | $-G$ | - | $0$ |
| 3-Invest. in fixed K[13] | - | $+p\cdot\Delta K$ | $-p\cdot\Delta K$[14] | - | - | $0$ |
| 4-Accounting Memo (1): “Final” Sales at market prices $\equiv p\cdot X = C + G + p\cdot\Delta K \equiv W + FT \equiv Y$ | | | | | | |
| 5-Wages | $+W$ | $-W$ | - | - | - | $0$ |
| 6-Taxes | $-Tw$ | $-Tf$ | - | $+T$ | - | $0$ |
| 7-Interest on Loans | - | $-i_{-1}\cdot L_{-1}$ | - | - | $+i_{-1}\cdot L_{-1}$ | $0$ |
| 8-Interest on Bills | - | - | - | $-ib_{-1}\cdot B_{-1}$ | $+ib_{-1}\cdot B_{-1}$ | $0$ |
| 9-Interest on Deposits | $+ib_{-1}\cdot D_{-1}$ | - | - | - | $-ib_{-1}\cdot D_{-1}$ | $0$ |
| 10-Dividends | $+Ff + Fb$ | $-Ff$ | - | - | $-Fb$ | $0$ |
| 11-Column Totals | $SAVh$ | $Fu$ | $-p\cdot\Delta K$ | $SAVg$ | $0$ | $0$ |

If it is true that beginning of period stocks necessarily affect income flows (and, as we shall see, asset prices), it is also true that saving flows and capital gains necessarily affect end of period stocks[15]. This is shown in table 3. Given the hypotheses above, households' saving necessarily implies changes in their holdings of bank deposits and/or stocks, while government deficits are necessarily financed with the emission of T-bills,

[13] We follow here the broad Keynesian literature in simplifying away investment in inventories (which plays a crucial role in Godley and Cripps, 1983 and Shaikh, 1989). These can be easily introduced (say, along the lines of Godley, 1999), though only at the cost of increasing the complexity of the model.

[14] Firms' investment expenditures in physical capital imply a change in their financial or capital assets and, therefore, is a “capital” transaction. As such it (re)appears in table 3 below. The reason it is included in table 2 is to stress the idea that firms buy their capital goods from themselves, an obvious feature of the real world (though a slightly odd assumption in our “one good economy”) .

[15] Fluctuations in the price of the single good produced in the economy (for firms) and in the market value of equities (for firms and households) are the only sources of nominal capital gains and losses in this economy.

and investment is necessarily financed by a combination of retained earnings, equity emissions and bank loans. As emphasized by Godley (1999), the banking sector plays a crucial role in making sure these inter-related balance sheet changes are mutually consistent[16].

| Table 3: Flows of Funds in our artificial economy<br>Positive figures denote sources of funds, while negative ones denote uses of funds. | | | | | |
|---|---|---|---|---|---|
| | Households | Firms | Banks | Govt. | Row Totals |
| Current Saving | $+SAVh$ | $+Fu$ | $0$ | $+SAVg$ | $+SAV$ |
| Δbank deposits | $-\Delta D$ | | $+\Delta D$ | | $0$ |
| Δloans | | $+\Delta L$ | $-\Delta L$ | | $0$ |
| ΔTreasury Bills | | | $-\Delta B$ | $+\Delta B$ | $0$ |
| Δcapital | | $-p\cdot\Delta K$ | | | $-p\cdot\Delta K$ |
| Δequities | $-\Delta E\cdot pe$ | $+\Delta E\cdot pe$ | | | $0$ |
| Column Totals | $0$ | $0$ | $0$ | $0$ | $0$ |
| Δnet Worth (Accounting Memo) | $SAVh + \Delta pe\cdot E_{-1}$ | $Fu + \Delta p\cdot K_{-1} - \Delta pe\cdot E_{-1}$ | $0$ | $SAVg$ | $SAV + \Delta p\cdot K_{-1} \equiv p\cdot\Delta K + \Delta p\cdot K_{-1}$ |

Before we continue, we must remind the reader that all accounts presented so far were phrased in nominal terms. All stocks and flows in tables 1 and 2 above have straightforward "real" counterparts given by their nominal value divided by $p$ (the price of the single good produced in the economy), while the "real" capital gains in equities are given by:

$\Delta pe_t\cdot E_{-1}/p_t - \Delta p\cdot pe_{-1}E_{-1}/p_{-1}\cdot p_t;$

and the "real" capital gains in any other financial asset $Z$ are given by:

$-\Delta p\cdot Z_{-1}/p_{-1}\cdot p_t$[17].

[16] As is well known, most macroeconomic models assume that some sort of Walrasian auctioneer takes care of financial intermediation. We believe this simplification is not faithful to the views of financially sophisticated post-Keynesians (such as Davidson, 1972; Godley and Cripps, 1983; and Minsky, 1986), though.

[17] Given that ours is a "one good" economy, the real value of physical capital is not affected by inflation.

## 2. A FIX-PRICE SIMPLIFIED POST-KEYNESIAN CLOSURE

We shall work here with a simplified "*fix-price*" version of the model, leaving the discussion of extensions to part 4.

### 2.1 Aggregate Supply

Following Taylor (1991, chapter 2), we assume that:

*(E1)* $p = w \cdot b \cdot (1+\tau)$;

where $p$ =price level, $w$ = money wage per unit of labor, $b$ = labor-output ratio, and $\tau$ = mark up rate. From (1) it is easy to prove that the (gross, before tax) profit share on total income ($\pi$) is given by:

*(E2)* $\pi = [p \cdot X - W]/p \cdot X = \tau/(1+\tau)$;

so that the (before tax) wage share on total income is:

*(E3)* $1 - \pi = W/ p \cdot X = 1/(1+\tau)$;

and $W = (1 - \pi) \cdot p \cdot X$.

We assume here that the price level, the technology, and the income distribution of the economy are exogenous, so all lower case variables above are constant (i.e. the aggregate supply of the model is horizontal)[18].

### 2.2 Aggregate Demand

*2.2.1 A "Kaleckian SFC" Consumption Function.*

The hypothesis here is that production workers spend all they get after taxes, while "capitalist households" spend a fraction of their (lagged) wealth (as opposed to their current income, as in Kalecki)[19]. The rationale for this simplification is the idea that rich people are more concerned with their wealth than with their income[20]. Formally,

*(E4a)* $C = W - Tw + a \cdot Vh_{-1} = W \cdot (1-\theta) + a \cdot Vh_{-1}$;

[18] All these assumptions can be relaxed, of course, provided one is willing to pay the price of increased analytical complexity.

[19] The term "capitalist households" is used here to designate rentiers and managerial workers. In other words, the wages of managerial workers are assumed to be a part of the distributed profits of firms.

[20] More realistic assumptions could easily be introduced, though only at the cost of making the algebra considerably heavier.

where $\theta$ is the income tax rate and $a$ is a fixed parameter[21]. Following Taylor (1991), we normalize the expression above by the (lagged) value of the stock of capital[22] to get:

*(E4)* $C/p\cdot K_{-1} = (1 - \pi)\cdot(1-\theta)\cdot u + a\cdot vh_{-1};$

where $u = X/K_{-1}$, and $vh_{-1} = Vh_{-1}/ p\cdot K_{-1}$.

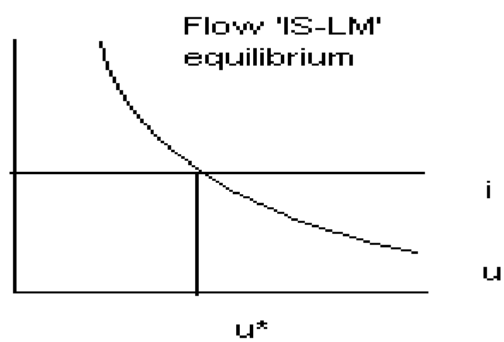
*2.2.2 A "Structuralist" Investment Function*:

The simplest version of the model presented here uses Taylor's (1991, chapter 5) "structuralist" investment function (which, in turn, is an extension of the one used in Marglin and Bhaduri, 1990)[23]:

*(E5)* $g^i(\pi, u, i) = go + (\alpha\cdot\pi + \beta)\cdot u - \theta_1\cdot i;$

where $g^i(\pi, u, i) = \Delta K/ K_{-1}$, $i$ is the interest rate on loans, and $go$, $\alpha$, $\beta$, and $\theta_1$ are exogenous parameters measuring the state of long term expectations ($go$), the strength of the "accelerator" effect ($\alpha$ and $\beta$), and the sensibility of aggregate investment to increases in the interest rate on bank loans ($\theta_1$). In part 4 we discuss what happens when one modifies this investment function along the lines suggested by Lavoie and Godley (2001-2002).

*2.2.3 – The "u" Curve*




[21] As discussed below, we assume that $Tw = \theta\cdot W$ (i.e. that capitalist households' income is not taxed).

[22] Taylor uses the current stock of capital because he works in continuous time. As both the formalization and the checking (through computer simulations) of stock-flow consistency requirements are reasonably complex in continuous time (and no proportional insight appears to be added), we work here in discrete time and assume (as Keynes) that the stock of capital available in any given "short period" is pre-determined (i.e. that investment does not translate into capital instantaneously).

[23] Though, as noted by Foley and Taylor (2004, p.2), we could easily have assumed also a "Harrodian" (or "Classical") specification in which investment demand would adjust gradually to stabilize long-run capacity utilization (as proposed, among others, by Shaikh, 1989 and Godley, 1996). On the same vein, Skott (1989) provides a possible rationalization for the "desired" stock (of physical capital)—flow (of final sales) of firms.

Assuming that both $\gamma = G/ p \cdot K_{-1}$ and $i$ are given by policy, the ("short period") goods' market equilibrium condition is given by:

$p \cdot X = W \cdot (1-\theta) + a \cdot Vh_{-1} + [go + (\alpha \cdot \pi + \beta) \cdot u - \theta_1 \cdot i] \cdot p \cdot K_{-1} + \gamma \cdot p \cdot K_{-1}$;

or, after trivial algebraic manipulations,

(E6) $u = [a \cdot vh_{-1} + go - \theta_1 \cdot i + \gamma] / [1 - (1 - \pi) \cdot (1-\theta) - (\alpha \cdot \pi + \beta)]$;

which is essentially the normalized "IS" curve of the model. In fact, the "short period" equilibrium of the model has a straightforward "IS-LM" (of sorts) representation, which implies that "short period" comparative static exercises can be done quite simply (more on this below). Note finally that, the (temporary, goods' market) equilibrium above only makes economic sense if the sum of the propensity to consume out of current income [i.e. $(1-\pi) \cdot (1-\theta)$] and the "accelerator" effect [i.e $\alpha \cdot \pi + \beta$] is smaller than one.

## 2.3 Financial Behavior and Markets

Up until now, the model is very similar to, say, "modern" "new Keynesian consensus" ones. Indeed, even though neither a Philips' curve relation nor a "monetary policy rule" were assumed, we could easily close the model along these lines. Contrarily to "new consensus" models, however, our IS equation depends on the distribution of income and on capitalist households' stock of wealth. Moreover, we do not ignore/trivialize the financial structure of the economy, so we still have a lot of ground to cover.

### *2.3.1 – Financial Behavior of Households*

The two crucial hypotheses here are that: (i) households make no expectation mistakes concerning the value of $Vh$[24], and (ii) the share $\delta$ of equity (and, of course, the share $1- \delta$ of deposits) on total household wealth depends negatively (positively) on $ib$ and positively (negatively) on the expectational parameter $\rho$. Formally we have that:

(E7) $pe \cdot E^d = \delta \cdot Vh$;

(E8) $D^d = (1 - \delta) \cdot Vh$; and

(E9) $\delta = -ib + \rho$;

[24] The inclusion of expectation errors (say, along the lines of Godley, 1999), would imply the inclusion of hypotheses about how households react to these errors, making the model "heavier."

where $\rho$ is assumed to be constant in this simplified "closure"[25]. The value of $Vh$, on the other hand, is given by the households' budget constraint (see table 3 above):

$Vh \equiv Vh_{-1} + SAVh + \Delta pe \cdot E_{-1}$ ;

while from table 2 and (E4), it is easy to see that $SAVh = ib_{-1} \cdot D_{-1} + Ff + Fb - a \cdot Vh_{-1}$,

so that:

(E10) $Vh = (1 - a) \cdot Vh_{-1} + ib_{-1} \cdot D_{-1} + Ff + Fb + \Delta pe \cdot E_{-1}$.

*2.3.2 – Financial Behavior of Firms*

For simplicity, we assume that firms keep a fixed $E/K$ rate ($\chi$) and distribute a fixed share $\mu$ of its (after-tax, net of interest payments) profits[26], so that:

(E11) $E^s = \chi \cdot K = \chi \cdot K_{-1} \cdot (1+g^i)$;

(E12) $Ff = \mu \cdot [(1-\theta) \cdot \pi \cdot u \cdot p \cdot K_{-1} - i_{-1} \cdot L_{-1}]$; and

(E13) $Fu = (1 - \mu) \cdot [(1-\theta) \cdot \pi \cdot u \cdot p \cdot K_{-1} - i_{-1} \cdot L_{-1}]$.

And, as the price of equity ($pe$) is supposed to clear the market, we have also that:

(E14) $E^s = E^d$; so that (from E7 and E11):

(E15) $pe = \delta \cdot Vh/(\chi \cdot K)$.

Firms' demand for bank loans, in turn, can be obtained by replacing (E5), (E11), and (E13) in their budget constraint (see table 3). Indeed, from

$\Delta L \equiv p \cdot \Delta K - pe \cdot \Delta E - Fu$, it is easy to see that (assuming that $\chi = \chi_{-1}$):

(E16) $L^d = (1 + i_{-1} - \mu \cdot i_{-1}) \cdot L_{-1} + g^i \cdot p \cdot K_{-1} - pe \cdot g^i \cdot \chi \cdot K_{-1} - (1-\mu) \cdot (1-\theta) \cdot \pi \cdot u \cdot p \cdot K_{-1}$

*2.3.3 – Financial Behavior of Banks and the Government*

For simplicity, banks are assumed here (*a la* Lavoie and Godley, 2001-2002) to provide loans as demanded by firms[27]. In fact, banks' behavior is essentially passive in the simplified model discussed here, for we also assume that (i) banks always accept

[25] Though it plays a crucial role in Taylor and O'Connel's (1985) seminal "Minskyan" model. The simplified specification above is perhaps an extreme version of Keynes' (1997 [1936], p.154) view that the demand for equities "(…)is established as the outcome of the mass psychology of a large number of ignorant individuals (..)" and, therefore, is "liable to change violently as the result of a sudden fluctuation in opinion due to factors that do not really much make difference to the prospective yield (…)". Specifications connecting $\delta$ to expected dividends (as, for example, the one in Zezza and Dos Santos, 2004) could also have been used, of course, though only at the cost of making the algebra heavier.

[26] Varying $\chi$ and $\mu$ can be easily introduced, though only at the cost of making the algebra heavier. Note, however, that the hypothesis of a relatively constant $\chi$ is roughly in line with the influential New-Keynesian literature on "equity rationing" (see Stiglitz and Greenwald, 2003, chapter 2 for a quick survey).

[27] We discuss a "credit crunch" regime in part 4.

deposits from households and T-bills from the government, (ii) banks distribute whatever profits they make[28], and (iii) the interest rate on loans is a fixed mark up on the interest rate on T-bills. Formally:

(E17) $L^s = L^d = L;$

(E18) $D^s = D^d = D;$

(E19) $Bb^d = B^s = B;$

(E20) $i = (1+ \tau_b)\cdot ib,$ and

(E21) $Fb = i_{-1}\cdot L_{-1}+ ib_{-1}\cdot Bb_{-1}- ib_{-1}\cdot D_{-1}.$

The behavior of the government sector is also very simple. Its taxes are a fixed proportion of wages and gross profits, its purchases of goods are a fixed proportion of the (lagged) stock of capital, its supply of T-bills is given by its budget constraint (see tables 2 and 3 above), and the interest rate on T-bills is whatever it decides it is. Formally,

(E22) $G = \gamma\cdot p\cdot K_{-1};$

(E23) $T = Tw + Tf = \theta\cdot W + \theta\cdot(p\cdot X - W) = \theta\cdot p\cdot X;$

(E24) $B^s = (1 + ib_{-1})\cdot B_{-1} + \gamma\cdot p\cdot K_{-1} - \theta\cdot p\cdot X;$ and

(E25) $ib = ib^*.$

## 3. COMPLETE "TEMPORARY" AND "STEADY STATE" SOLUTIONS

One of the methodological advantages of the SFCA is that it allows for a natural integration of "short" and "long" periods. In particular, both Keynesian notions of 'long period equilibrium' and "long run" acquire a precise sense in a SFC context, the former being the steady-state equilibrium of the stock-flow system (assuming that all parameters remain constant through the adjustment process), and the latter being the more realist notion of a path-dependent sequence of 'short periods' in which the parameters are subject to sudden and unpredictable changes. These concepts are discussed in more detail in section 3.2 below. Before we do that, however, we need to discuss the characteristics of the "short period" (or "temporary") equilibrium of the model.

[28] Under this assumption allowing banks to hold a fraction $\delta^*$ of its deposits in equities is one and the same thing of adding $\delta^*$ to $\delta$ (hence our hypothesis that only households buy equities). Assuming that the banks' net worth can differ from zero would only make the algebra considerably more complex, however.

### 3.1 The "Short Period" Equilibrium

In any given (beginning of) period, the stocks of the economy are given, inherited from history. The solution of the model under these hypotheses is discussed in section 3.1.1 below, while an intuitive numerical example is discussed in section 3.1.2.

*3.1.1 – The Analytics of the "Short Period" Equilibrium*

As discussed in section 2.2.3 given $Vh_{-1}$, $K_{-1}$, distribution parameters, and fiscal and monetary policy, the (normalized) level of economic activity is trivially given by:

(E6) $u = [a \cdot vh_{-1} + go - \theta_I \cdot i + \gamma] / [1 - (1 - \pi) \cdot (1-\theta) - (\alpha \cdot \pi + \beta)]$ [29].

But (demand-driven) economic activity is hardly the only variable determined in any given "period." Equally important are the stock (i.e. balance sheet) implications of the sectoral income and expenditure flows and portfolio decisions (for end of period stocks necessarily affect income flows in the next period). Fortunately, it is straightforward to prove (see appendix) that the (normalized) end-of period financial stocks can be written as:

(E26) $b = [b_{-1} \cdot (1+ib_{-1}) + \gamma - \theta \cdot u ] / (1 + g^j);$

(E27) $vh = [\psi 1 \cdot vh_{-1} + (1-\theta) \cdot \mu \cdot \pi \cdot u + \psi 2 \cdot b_{-1}] / (1 + g^i - \delta);$

(E28) $d = (1- \delta) \cdot vh,$

(E29) $l = d - b;$ and

(E30) $vf = 1 - \delta \cdot vh - l;$

where,

$\psi 1 = [1 - a - \delta + (1- \mu) \cdot (1 - \delta) \cdot i_{-1}];$

$\psi 2 = ib_{-1} - (1-\mu) \cdot i_{-1};$

and *b, vh, d, l,* and *vf* stand for their uppercase counterparts normalized by the (current) value of the capital stock (i.e. $l = L/p \cdot K$; $l_{-1} = L_{-1}/p \cdot K_{-1}$; and so on).

Now note that if *b* and *vh* are known, then *d, vf* and *l* are easily determined by equations (E28)-(E30) above. As a consequence, the solution of the model can be represented by the following (*buv*) system:

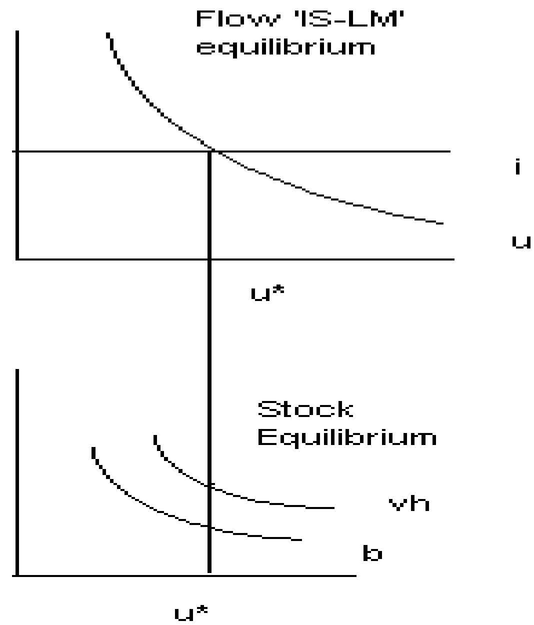
(E26) $b = [b_{-1} \cdot (1+ib_{-1}) + \gamma - \theta \cdot u ] / (1 + g^j);$

(E6) $u = [a \cdot vh_{-1} + go - \theta_I \cdot i + \gamma] / [1 - (1 - \pi) \cdot (1-\theta) - (\alpha \cdot \pi + \beta)];$

[29] Assuming, naturally, that the economy is below full capacity utilization. The discussion of inflation is postponed to part 4.

$(E27)\ vh = [\psi 1 \cdot vh_{-1} + (1-\theta) \cdot \mu \cdot \pi \cdot u + \psi 2 \cdot b_{-1}] / (1 + g^i - \delta);$

so the temporary equilibria of the system has the clear-cut graphic representation below[30],




and, again, the (period) comparative statics exercises are straightforward. As a matter of fact, the model admits a convenient recursive solution. Given u (which can be calculated directly from the initial stocks, monetary and fiscal policies and distribution and other parameters), one can easily get $b$ and $vh$ and, given these last two variables, one can then calculate $l$, $d$ and $vf$ (and, therefore, $q$[31]).

### *3.1 2 A Numerical Example*[32]

Despite the unfriendly appearance of the algebra above, the functioning of the artificial economy it describes is expected to be fairly intuitive to anyone familiar with the Keynesian tradition. Suppose, for example, the given initial stocks and set of parameters and initial conditions (and make $p = 1$):

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[30] The positions of the *buv* curves are determined by history and, therefore, change every period. We note, however, that the $vh$ curve will be higher then the $b$ curve in all relevant cases. To see that, one must first note (from consolidating the balance sheets in table 1) that $b + 1 \equiv vh + vf$. Since the maximum (relevant) value of $vf$ is 1 (assuming that both loans and the price of equity go to zero, and firms do not accumulate financial assets), it is easy to see that $vh$ has to be bigger than $b$. The appendix discusses the slopes of the $l$, $b$, and $vh$ curves.

[31] For Tobin's $q \equiv 1 - vf$

[32] This section follows Godley and Cripps's (1983) approach to numerical simulation very closely.

| | Households | Firms | Banks | Government + Central Bank | Row Totals |
|---|---|---|---|---|---|
| 1-Central Bank Advances | | | *0* | *0* | *0* |
| 2-Bank Deposits | *+170* | | *-170* | | *0* |
| 4-Bank Loans | | *-170* | *+170* | | *0* |
| 5-Bills | | | *0* | *0* | *0* |
| 6-Capital Goods | | *+200* | | | *+200* |
| 7-Equities | *+30* | *-30* | | | *0* |
| 8-Net Worth (Column Totals) | *200* | *0* | *0* | *0* | *+200* |

| $X$ | $i$ | $ib$ | $\gamma$ | $go$ | $\alpha$ | $\beta$ |
|---|---|---|---|---|---|---|
| 1 | 0.05 | 0.03 | 0.13 | 0.015 | 0.3 | 0 |
| $\theta_1$ | $a$ | $\mu$ | $\theta$ | $\pi$ | $\delta$ | $pe$ |
| 0.1 | 0.03 | 0.75 | 0.25 | 0.2 | 0.15 | .15 |

A stylized story of what "happens" in any given period would go as follows:

1 – In the classic "circuitist" tradition (e.g. of Graziani, 2003), firms are assumed to get bank loans to finance production (in the beginning of the period). Since they are assumed to get the point of effective demand right they get a \$100 loan. In fact, from (E6) it is easy to prove that:

$p\cdot X = u\cdot p\cdot K_{-1}$=[.03·1 +.015 –(.1·.05) + .13] /[1 - (1 – .2)·(1-.25)- (.03·.2 + 0)] ·200= 100.

Now the banks' balance sheet consists of bank loans to firms of \$270 (i.e. 170+100) and firms' deposits of 100 (0+100) plus households' deposits of 170.

2 – Firms pay wages of \$80 (.8·100*)* with bank checks, so the firms' bank deposits are now \$20 (100-80), while households' deposits reach \$250 (170 + 80).

3 – Households spend money in consumption goods (0.75·80 + .03·200 = \$66) and pay their taxes (0.25·80= \$20) using bank checks. So households' deposits go down (to \$164 = 250-66 -20), while firms' loans go down to \$204 (270 - 66) as firms use their receipts to pay down their debt with banks. Finally, government deposits (in the amount of \$20) are created.

4 – The government buys goods from firms (.13·200 = \$26) using bank checks and pays the service of its debt to banks (0.05·0= \$0), so that its deposits go to zero and it

has to give \$6 in T-bills to banks. The firms again use the receipts to pay down their debt, which therefore goes down to \$178 (204– 26).

5 - Firms use their deposits (\$20) to pay their profit taxes (\$5 = 0.25·20) to the government, their debt service (\$8.50 = 0.05·170) to banks, and distribute profits (\$4.88 = 0.75·(20 - 5 - 8.50)) to households, using retained earnings (\$1.62 = 0.25·(20 - 5 - 8.50)) to cut down their loans (to \$176.38 = 178 – 1.62). The government uses the firms' tax money to buy back (\$5 in) government bills from banks (cutting down its debt to \$1 = 6-5), while households' deposits reach \$177.38 (164 + 3.40 + 5.10 + 4.88), increased by interest payments on their deposits (\$3.40) and distributed profits of banks (\$5.10) and firms (\$4.88).

6 - Firms get money from selling new equity to households. To know exactly how much firms make selling new equity note that the equilibrium in the stock market happens when $pe = .15 \cdot Vh/(\chi \cdot K)$ (E15). Note also that since $\chi = 1$, we have that $E = K = 200 \cdot (1 + gi)$. But we know (from (E5), $u$, and the parameters above) that $gi = .04$, so $K = 208$. From these two facts one can conclude that $pe = .15 \cdot Vh / 208$. Finally, from the budget constraint of households (see tables 2 and 3 above) we know that $Vh = Vh_{-1} + SAVh + \Delta pe \cdot E_{-1}$, so that $Vh$ = \$200 + \$7.38 (= $SAVh$ = 177.38 - 170 = the increase in households' bank deposits so far)+ $.15 \cdot Vh \cdot 200/208$ – \$30, or equivalently, $Vh \approx 207.2$, so that $pe$ drops to 0.149. As a consequence, firms get \$1.19 (≈ 0.149*8) from households in new equities. These purchases allow firms to reduce their loans to \$175.19 (= 176.38 – 1.19), while simultaneously reducing households' deposits to \$ 176.19 (= 177.38 – 1.19). Interestingly enough, firms' retained earnings (\$1.62) and the fall in stock prices combined to cause the net worth of firms to get positive (\$1.8 ≈ 208 – 175.19 - .149·208).

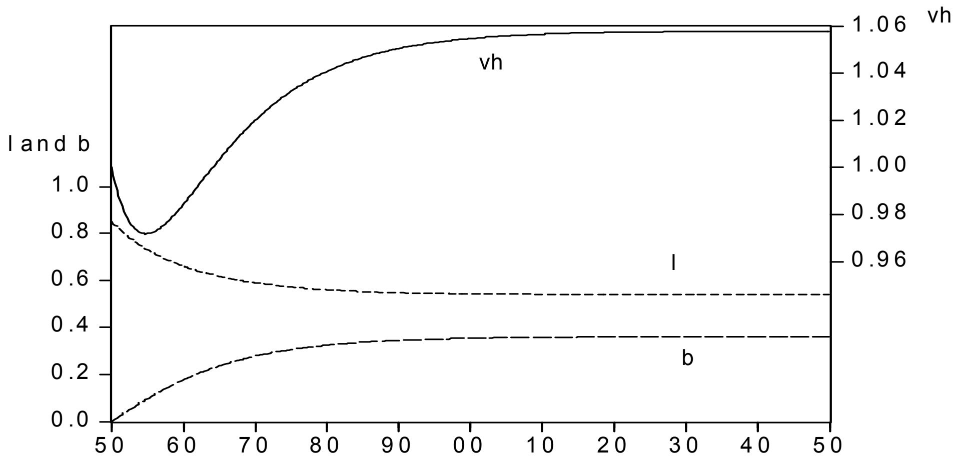
So, in the end of period 1, one has the following new balance sheets:

| | Households | Firms | Banks | Government + Central Bank | Row Totals |
|---|---|---|---|---|---|
| 1-Central Bank Advances | | | *0* | *0* | *0* |
| 2-Bank Deposits | *+176.19* | | *-176.19* | | *0* |
| 4-Bank Loans | | *-175.19* | *+175.19* | | *0* |
| 5-Bills | | | *+1* | *-1* | *0* |
| 6-Capital Goods | | *+208* | | | *+208* |
| 7-Equities | *+31* | *-31* | | | *0* |
| 8-Net Worth (Column Totals) | *207.19* | *1.8* | *0* | *-1* | *+208* |

### 3.2 The "Long Period" (i.e. Steady-State) Equilibrium and Its Interpretation

The "*buv*" system also allows one to understand what would happen in the artificial economy described above if its parameters would remain constant through a sufficiently big number of periods. As we saw above, both the capacity utilization and the (normalized) balance sheets of the economy are completely determined by policy distribution and behavioral parameters (which are all, by hypothesis, constant) and the (normalized) beginning of period stocks of household wealth and public debt. Under a given set of circumstances, the stock-flow system described above will converge (at a speed determined by its parameters) to a "long period" steady-state in which both $u$, $pe$ and the normalized balance sheets of the economy are constant. All one has to do to calculate this "long period equilibrium" is to solve the *buv* system above under the assumptions that $vh = vh_{-1} = vh^*$ and $l = l_{-1} = l^*$ (see the appendix, for a discussion). In the case of the numerical example given above, for instance, the system converges to its steady state after approximately 200 quarters (see below)[33].

[33] That is to say, after 200 periods like the one described in the example above have come to pass. For scaling reasons, the dynamics of $u$ would not be clear in the graph above. From (E6), however, we know that $u$ is a linear function of $vh_{-1}$, so it's easy to conclude that $u$ and $vh$ follow similar dynamic paths.




Naturally enough, no one expects the economy to eventually reach this steady-state, for the very good reason that no one expects its parameters to remain constant for 200 quarters[34]. Having a *ceteris paribus* idea of where the economy is heading is an important input in assessing the likelihood of future changes, though. If, say, one notes that the loan to capital ratio is growing without bounds or is tending to a very high level, he or she will have every reason to suspect that some structural or parametric change will happen in the system to prevent these outcomes[35].

To be sure, in the Post-Keynesian world described above a lot of things are expected to change every single period. Expectations, for example, are assumed to affect both the investment function and the portfolio choice of households (and, therefore, the financial conditions of firms and households). The economy can easily find itself (or be put) in unsustainable situations (i.e. those in which the steady-state is not stable or implying very high, or low, stock-flow ratios[36]), e.g. if the government fixes the interest rate on public debt higher than the growth rate of the economy (as it is often the case in Latin America), or if enthusiastic entrepreneurs force the loans to capital ratio to very high levels (as Minsky, 1982, 1986, would have put it). We believe the framework above

[34] Though note that considerably faster adjustments can happen.

[35] As noted by Godley and Cripps (1983, p. 42) it is reasonable to assume that "stock variables will not change indefinitely as ratios to the related flow variables." In the same vein, Minsky (e.g. 1982) used stock (of debt and liquid assets)-flow of (disposable income) ratios as proxies for the "financial fragility" of institutional sectors.

[36] The stability of the *buv* system cannot, of course, be taken for granted. The system is highly non-linear for several reasons, one of which being that "regime changes" necessarily happen whenever a stock reaches zero.

provides a simple formal way to tell these and other classical Structuralist/Post-Keynesian path-dependency “long run” stories[37].

## 4 A BRIEF DISCUSSION OF EXTENSIONS AND SIMPLIFICATIONS OF THE MODEL

Whether or not the model presented above achieves a good blend of realism and simplicity is debatable. This section aims to help the reader to form an opinion discussing possible extensions and a major simplifying assumption—i.e. the hypothesis that firms’ net worth is zero (associated with the Modigliani-Miller, 1958, theorem).

*4.1.1 – More Complex Investment and Consumption Functions*

Keynesians often assume that $g^i$ is a function of a number of financial variables. Lavoie and Godley (2001-2002, L&G from now on), for example, propose to include Tobin’s $q$, the loan to capital ratio and the retained earnings to capital ratio as determinants of $g^i$. While we do prefer to model credit constraints in the supply of loans (see section 4.1.2 below), we note that the *buv* structure above is robust to the adoption of a L&G’s specification. To see this let us assume, *a la* L&G, that:

(E5a) $g^i(\pi, u, i) = go + (\alpha \cdot \pi + \beta) \cdot u + \eta 1 \cdot q_{-1} - \eta 2 \cdot i \cdot l_{-1} + \eta 3 \cdot Fu/p \cdot K_{-1};$

where $q = (pe \cdot E + L)/p \cdot K$ is Tobin’s $q$ and $\eta 1, \eta 2, \eta 3$ are fixed positive parameters.

Now, from (E7) and (E13) it is easy to show that:

$g^i(\pi, u, i) = go + \varphi 1 \cdot u + \varphi 2 \cdot l_{-1} + \eta 1 \cdot \delta \cdot vh_{-1}$;

where $\varphi 1 = \alpha \cdot \pi + \beta + \eta 3 \cdot (1 - \mu) \cdot (1 - \theta) \cdot \pi$, and $\varphi 2 = \eta 1 - \eta 2 \cdot i - \eta 3 \cdot (1 - \mu) \cdot i_{-1}$.

In other words, L&G’s specification only adds on $l_{-1}$ (i.e. $(1 - \delta_{-1}) \cdot vh_{-1} - b_{-1}$) to the determinants of $u$ and causes the latter to depend on $vh_{-1}$ in a more complex way than before. In sum, the new specification only makes the *buv* system more complex. The same reasoning applies also to a wide range of consumption functions, as long as the

[37] Taylor (2004, p. 258) notes that “the trouble with most macroeconomic models of finance is that they don’t let anything interesting happen.” We tried to make sure that this is not the case here.

propensity to save out of wealth is the same for both capitalist and production workers[38]. It is easy to see also that the inclusion of current values of, say, $q$ in the investment function or $vh$ in the consumption function would change the *buv* curves in such a way that the system no longer would admit the recursive solution discussed above.

### *4.1.2 The "Credit Crunch" Regime and Minskyan crises*

It is also common in the Keynesian literature to find the hypothesis that the maximum supply of bank loans to firms depends on the interest rate on loans, on the business cycle and on the financial fragility of firms[39]. If this is the case, and assuming (as Stiglitz and Greenwald, 2003) that the interest rate on loans does not "clear" the market[40], i.e. that:

$$L^d = (1 + i_{-1} - \mu \cdot i_{-1})\cdot L_{-1} + g^i\cdot p\cdot K_{-1} - pe\cdot g^i\cdot\chi\cdot K_{-1} - (1-\mu)\cdot\pi\cdot u\cdot p\cdot K_{-1} > L^s(i, u, l_{-1});$$

it is easy to show that $g^i$ has to adjust to make sure that $L^d = L^s$. Formally, this means replacing (E5) above by:

$$\text{(E5b)}\quad g^i = [l^s - (1 + i_{-1} - \mu \cdot i_{-1})\cdot l_{-1} + (1-\mu)\cdot\pi\cdot u] / (1 - l^s - \delta\cdot vh)$$

and, of course, replacing (E6), (E27), and (E29) by

$$\text{(E6a)}\quad u = \{a\cdot vh_{-1} + \gamma + [l^s - (1 + i_{-1} - \mu\cdot i_{-1})\cdot l_{-1}]/(1 - l^s - \delta\cdot vh)\}/\{\theta\cdot(1-\pi) + \pi - [(1-\mu)\cdot\pi/(1 - l^s - \delta\cdot vh)]\};$$

$$\text{(E27a)}\quad vh = (l + b)/(1-\delta);$$ and

$$\text{(E29a)}\quad l = l^s(i, u, l_{-1});$$ so the *buv* system becomes a *blu* one.

It so happens that replacing (E5b) in the goods' market equilibrium condition causes $u$ to depend in a rather complex way on current and lagged values of $vh$ and $l$ *(E6a)*, so that the (now) *blu* system no longer admits a simple recursive solution. This "credit crunch" story seems to us at least as faithful to Minsky's (and Keynes's) writings as the ones told by the so-called "formal Minskyan literature" (Dos Santos, 2004a), which usually place the burden of producing Minskyan crises on the investment specification[41].

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[38] If workers consume relatively more or less of their wealth than capitalists, we would have to disaggregate household wealth in capitalists' and workers' wealth, introducing an additional stock variable to the model. The same happens if we want to incorporate household debt into the analysis.

[39] See, among others Stiglitz and Greenwald (2003) and Delli Gatti et al. (1994).

[40] For in that case the interest rate on loans would be endogenous, fluctuating to make $L^d = L^s$.

[41] While "Minskyan results" can be caused by fluctuations in both lenders' and borrowers' risks, the "formal Minskyan literature" (initiated by Taylor and O'Connel, 1985) has either emphasized the latter over the former or worked with "reduced form" investment functions in which both are described by the same variables. The "credit crunch" regime above does the opposite, i.e. it depicts an extreme situation in which expectations are such that the "lenders' risk" curve becomes vertical. That both Minsky himself and

### 4.1.3 More Complex Financial Structures

What if the financial architecture discussed above is enriched to include high-powered money, holdings of T-bills by families and central bank advances to banks? Table 4 below depicts the balance sheets of this more complex economy[42].

Neglecting possible differences in taxation, the economy below differs from the one discussed in the previous sections in three major ways. First, it implies a more complex portfolio choice for households (who now have to choose among four instead of two assets) and a potentially more complex determination of capitalist households' income (given that the interest rates on deposits may differ from the interest rate on T-bills). Second it implies a more complex role for banks, which now can hold money and T-bills (being required by the central bank to hold a given minimum amount of high-powered money). Third, it (trivially) changes the budget constraint of the government, for now a part of the public debt is free of charge.

Table 4: Balance sheets of a more complex artificial economy

| | Households | Firms | Banks | Central Bank | Government | Row Totals |
|---|---|---|---|---|---|---|
| 1-High powered money | $+Hh$ | | $+Hb$ | $-H$ | | $0$ |
| 2-Central Bank advances | | | $-A$ | $+A$ | | $0$ |
| 3-Bank Deposits | $+D$ | | $-D$ | | | $0$ |
| 4-Bank Loans | | $-L$ | $+L$ | | | $0$ |
| 5-T-Bills | $+Bh$ | | $+Bb$ | $+Bc$ | $-B$ | $0$ |
| 6-Capital Goods | | $+p\cdot K$ | | | | $+p\cdot K$ |
| 7-Equities | $+E\cdot pe$ | $-E\cdot pe$ | | | | $0$ |
| 8-Net Worth (Column Totals) | $+ Vh$ | $+ Vf$ | $0$ | $0$ | $-B$ | $+p\cdot K$ |

Keynes were well aware of this latter possibility is clear in Minsky (1975, p.119). See also Keynes (1937, p.668-9).

[42] A detailed discussion of such an economy, and of how several schools of Keynesian thought rationalized its behavior, can be found in Dos Santos (2004b).

To see what is implied by the first two changes, consider the (Zezza and Dos Santos, 2004) case in which:

$pe \cdot E^d = \delta_1 \cdot (Vh - Hh^d)$;

$D^d = \delta_2 \cdot (Vh - Hh^d) = D^s = D$;

$Bh^d = (1 - \delta_1 - \delta_2) \cdot (Vh - Hh^d) = Bh$;

$Hh^d = a_1 \cdot u \cdot p \cdot K_{-1} = Hh$; and

$Hb^d = a_2 \cdot D = Hb$

It is now easy to see that the model now implies more complex specifications for both *pe* and the *SAVh* and *Vh* functions. Beginning with the first, note that the new stock market equilibrium condition is:

$pe \cdot E^d = \delta_1 \cdot (Vh - Hh^d) = pe \cdot E^s = pe \cdot \chi \cdot K$;

so that $pe = (p \cdot \delta_1 \cdot vh/\chi) - [\delta_1 \cdot a_1 \cdot u \cdot p/\chi \cdot (1 + g^j)]$[43].

Moreover, banks' stock of loanable funds is reduced in this economy, for two basic reasons: (i) households keep a part of their non-equity wealth in T-bills (so the amount of bank deposits gets smaller); and (ii) banks are required to hold a fraction *a2* of their total deposits *D* in high-powered money. So, the relevant equation for *Bb* becomes:

$Bb = Max\ [0, (1-a2) \cdot D - L]$, and, of course,

$A = -\ Min\ [0, (1-a2) \cdot D - L]$[44] (i.e. *A* is one and the same as a negative *Bb*).

As a consequence, we have now two "regimes" in the model, i.e. one in which *Bb* is positive and another in which *A* is positive (presumably increasing the likelihood of the credit crunch regime[45]). If the interest rate on central bank advances and T-bills differ, each of these regimes will imply a different *Fb* equation and, therefore, different *SAVh* and *Vh* equations. But given that *A* and *Bb* are straightforward functions of *Vh, L* and *u*, the model will still collapse to a *buv* system.

### *4.1.4 Inflation, Productivity and Distribution of Income*

As mentioned before, the "fix-price" algebra above assumes implicitly that capacity utilization does not reach its technical maximum. The model could easily incorporate a

[43] As opposed to $pe = (p \cdot \delta \cdot vh/\chi)$ in the simplified model.

[44] The specification above implies either that the interest on T-bills is smaller than the interest on central bank's advances or, if that is not the case, that the central bank monitors banks to ensure they do not get advances to finance purchases of T-bills.

[45] Especially if the central bank sets the interest rate on rediscount loans at punitive levels.

"forced savings" regime (as discussed by Taylor, 1991, p.47), as well as a wide range of (orthodox and heterodox) hypotheses about nominal wage, mark up and technical progress dynamics.

More importantly from our perspective, a serious treatment of inflation would require the model to be solved in "real" (i.e. deflated) terms. As discussed in section 1 above, this is straightforward for the flows but requires that the equations for all the financial stocks assumed above are changed to include the "real capital gains" formulas discussed in page 5[46]. Finally, it is quite obvious that ceteris paribus inflation will hurt creditors (i.e. households and banks) and benefit debtors (i.e. firms and the government). If large enough, it is likely to change the behavior of these sectors[47], though it should now be obvious to the reader that these changes will not hurt the general (and now "real") *buv* structure, only make it more complex.

*4.1.5 What if Vf = 0?*

In this case the *vh* function (and the *buv* system) gets considerably simpler. Indeed, consolidating the balance sheets in table 1 above one gets:

$Vh \equiv p \cdot K + B,$

or, equivalently,

$(E27b)\ vh \equiv 1 + b.$

But how can this new result be reconciled with our previous hypotheses? Essentially, the answer to this question is that $\delta$ now gets endogenous, so the household sector as a whole is supposed to always adjust their demand for equities to make sure it is paying exactly what the firms are worth. To see this formally, note first that from the firms' balance sheet we have that:

$p \cdot K \equiv L + pe \cdot E.$

Plugging the firms' loan demand (E16) in the identity above (and assuming $\chi = \chi_{-1}$), one gets:

$p \cdot K \equiv (1 + i_{-1} - \mu \cdot i_{-1}) \cdot L_{-1} + g^{i} \cdot p \cdot K_{-1} - pe \cdot g^{i} \cdot \chi \cdot K_{-1} - (1- \mu) \cdot (1-\theta) \cdot \pi \cdot u \cdot p \cdot K_{-1} + pe \cdot E;$

or, rearranging:

[46] So that the stock equations are altered. As the required changes are both simple, tedious, and do not affect the gist of our argument, we will not discuss them in this text.

[47] For instance, inducing households to buy equity as opposed to holding purely financial assets—the so-called "Tobin-effect" (Walsh, 1998, p.42).

$pe = (p/\chi)\cdot[1 - l_{-1}\cdot(1 - i_{-1} - \mu\cdot i_{-1}) + (1-\mu)\cdot(1-\theta)\cdot\pi\cdot u]$; so that

$\delta\cdot Vh = \delta\cdot(p\cdot K + B) = pe\cdot E = (p/\chi)\cdot[1 - l_{-1}\cdot(1 - i_{-1} - \mu\cdot i_{-1}) + (1-\mu)\cdot\pi\cdot u]\cdot E;$

or, after further rearranging:

$\delta = pe\cdot E/ Vh = p\cdot K\cdot[1 - l_{-1}\cdot(1 - i_{-1} - \mu\cdot i_{-1}) + (1-\mu)\cdot\pi\cdot u] / (p\cdot K + B).$

## 5 FINAL REMARKS

In the sections above we presented a simplified stock-flow consistent post-Keynesian growth model and related it to the existing (structuralist and post-Keynesian) literature(s)[48]. We are well aware that the specific derivations above depend crucially on the simplifying assumptions we made. We note, however, that the general *buv/blu* structure discussed above appears to be robust to wide changes in the flow specifications and/or financial architecture assumed—a point that, as far as we know, has not received enough attention in the aforementioned literatures.

Of course, little in this paper is theoretically new. Borrowing words from Foley and Sidrauski (1971, p. 6), our goal here was mostly to provide the reader with a rigorous and didactic "exposition of an eclectic tradition that strikes us as particularly coherent and logically convincing." It is up to the reader to decide whether or not we have succeeded.

[48] The relation of this kind of modeling with mainstream macroeconomics was discussed in Dos Santos (2004b) and, more generally, in Taylor (2004).

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## APPENDIX

As the u curve was derived earlier in the text, we begin by deriving the *b* and *vh* curves.

### A.1 – The *b* curve

From (E19) and (E24) we have that:

*(A.1.1)* $B = (1 + ib_{-1})\cdot B_{-1} + \gamma\cdot p\cdot K_{-1} - \theta\cdot p\cdot X;$

or, dividing everything by $p\cdot K_{-1}$,

*(A.1.2)* $b\cdot(1 + g^j) = (1 + ib_{-1})\cdot b_{-1} + \gamma - \theta\cdot u$,

where:

$b_{-1} = B_{-1}/(p\cdot K_{-1})$, and

$b = B/(p\cdot K)$.

It is now straightforward to see that:

*(E26)* $b = [b_{-1}\cdot(1+ib_{-1}) + \gamma - \theta\cdot u\,] / (1 + g^j)$[49].

### A.2 –The *vh* curve

From (E10), (E12) and (E21) we have that:

*(A.2.1)* $Vh = (1 - a)\cdot Vh_{-1} + \mu\cdot(1-\theta)\cdot \pi\cdot u\cdot p\cdot K_{-1} + (1-\mu)\cdot i_{-1}\cdot L_{-1} + ib_{-1}\cdot B_{-1} + \Delta pe\cdot E_{-1};$

Moreover, from (E11) and (E15) (and assuming that $\delta_{-1} = \delta$ and $\chi = \chi_{-1}$) we have that:

*(A.2.2.)* $\Delta pe\cdot E_{-1} = \delta\cdot Vh/(1 + g^j) - \delta\cdot Vh_{-1};$

and from the balance sheets of banks and (E8) we know that

*(A.2.3.)* $L_{-1} = (1 - \delta)\cdot Vh_{-1} - B_{-1}$;

So, replacing *(A.2.2)* and *(A.2.3)* in *(A.2.1)* and rearranging, we get:

*(A.2.4)* $Vh=[1-a-\delta+(1-\delta)\cdot(1-\mu)\cdot i_{-1}]\cdot Vh_{-1}+\mu\cdot(1-\theta)\cdot\pi\cdot u\cdot p\cdot K_{-1}+[ib_{-1}-(1-\mu)\cdot i_{-1}]\cdot B_{-1}+\delta\cdot Vh/(1+g^j)$

or, dividing both sides by $p\cdot K_{-1}$,

*(A.2.5)* $Vh/p\cdot K_{-1} = \psi 1\cdot vh_{-1} + \mu\cdot(1-\theta)\cdot \pi\cdot u + \psi 2\cdot b_{-1} + \delta\cdot vh;$

where:

$\psi 1 = [1 - a - \delta + (1-\mu)\cdot(1 - \delta)\cdot i_{-1}];$

$\psi 2 = ib_{-1} - (1 - \mu)\cdot i_{-1};$

$b_{-1} = B_{-1}/(p\cdot K_{-1})$;

$vh_{-1} = Vh_{-1}/(p\cdot K_{-1})$; and

[49] So that a necessary condition for the stability of the *buv* system is that $g > ib_{-1}$.

$vh = Vh/(p \cdot K)$.

As (by definition) $Vh/p \cdot K_{-1} = vh \cdot (1+g^i)$, it is now straightforward to show that:

$(E27)\ vh = [\psi 1 \cdot vh_{-1} + \mu \cdot (1-\theta) \cdot \pi \cdot u + \psi 2 \cdot b_{-1}] / \cdot (1+g^i - \delta)$[50].

**A.3 – The slope of the *b* and *vh* curves**

All one needs to do to get the slopes of the curves above (depicted in section 3.1.1) is to derive them with respect to $u$. It is intuitively clear, however, that $\partial b/\partial u < 0$ for increases in the economic activity generates increases in investment (and therefore capital growth) and in tax revenues, and both factors contribute to a smaller $b$. It so happens that $\partial vh/\partial u$ is also negative for a wide range of parameters and initial conditions. Indeed, increases in real activity have only a relatively modest positive impact on the stock of households' wealth (through its impact on the distributed profits of firms and, therefore, on households' income and saving) and affect the growth of the capital stock importantly (therefore affecting $vh = Vh/p \cdot K$ negatively)[51]. Finally, given that $b$ and $vh$ completely determine $l$, it is clear that $\partial l/\partial u$ also tends to be negative.

**A.4 – The steady-state solution of the *buv* system**

All one needs to do to get the steady state solution of the *buv* system is to solve it assuming that $vh = vh_{-1}$ and $b = b_{-1}$. It is then relatively easy to show that the characteristic equation of the system is cubic (what is unsurprising given the non-linearites in the *buv* system), potentially allowing for multiple (i.e. up to 3) "long run" equilibria. In the numeric example discussed in the text, however, two of the roots of the system were imaginary, so the steady-state solution depicted in section 3.2 was the only real one.

[50] So that a necessary condition for the stability of the *buv* system is that $\psi 1 / \cdot (1+g^i - \delta) < 1$, or equivalently that $g > (1-\mu) \cdot (1-\delta) \cdot i_{-1} - a$.

[51] This point can be seen formally. Indeed, it is easy to prove that $\partial vh/\partial u < 0$ when $vh > \mu \cdot (1-\theta) \cdot \pi / (\alpha \cdot \pi + \beta)$. In other words, the slope of $vh$ can only be positive when distributed profits are big and accelerator effects small.