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On Lower-bound Traps: A Framework for the Analysis of Monetary Policy in the "Age" of Central Banks

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Abstract: We present a simple theoretical framework that integrates the notion of the natural or neutral interest rate, liquidity preference theory, and the monetary policy practice by modern central banks. We claim that this theory explains the conditions under which an economy will experience an aggregate demand deficiency problem within a modern institutional setting. Contrary to the predictions of the New Consensus View in macroeconomics, the model suggests that "structural" factors such as a high saving rate and, especially, a low "natural" rate of growth increase the chances that an economy experiences an aggregate demand deficiency. Contrary to conventional wisdom, the model predicts that a fall in the NAIRU may lead to a rise in the natural interest rate, and vice versa.

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1 INTRODUCTION

A core proposition in the so-called "New Consensus View" (hereafter NCV) in macroeconomics holds that market economies possess strong self-regulation mechanisms which guarantee that any expansion of potential output generates an equi-proportional increase in the level of aggregate demand so that the latter adjusts passively to the former in the long run. The mechanism through which this adjustment process takes place is a crucial area of macroeconomic theory, yet it is frequently overlooked. In the Old Neoclassical Synthesis, such mechanisms used to come in the form of a real balance effect. However, a number of scholars have highlighted its practical irrelevance.[1] We will not discuss here the shortcomings of the real balance effect. We wish to note, though, that proponents of the NCV resort to another self-regulation mechanism. In particular, they assume that the aggregate demand function takes the form $y = -ar + g$, where y is the output gap—the difference between current output and potential output— r is the real interest rate, and g is a stochastic variable with zero mean capturing random shocks to aggregate spending, i.e., a shock to exports or fiscal policy (Clarida, Galí, and Gertler 1999; Taylor 2000).[2] This aggregate function implicitly poses that whenever a permanent shock raises potential output then aggregate demand increases in a similar magnitude. The mechanism through which this occurs is explained as follows:

[1] For instance, Greenwald and Stiglitz argue that "quantitatively, it is surely an *n*th order effect; one calculation put it that, even at the fastest rate at which prices fell in the Great Depression, it would take more than two centuries to restore the economy to full employment. And in the short run even its sign is ambiguous, as inter-temporal substitution effects may (depending on expectations) more than offset the wealth effects" (Greenwald and Stiglitz 1993). By contrast, Sims (2000) argues that "there is a potential for a large real balance effect in a deflationary environment and that its presence makes it extremely unlikely that we get into a liquidity trap." A discussion on the shortcomings of this effect is in Palacio-Vera (2005).

[2] A more sophisticated version of this function includes the expected future output gap (see Clarida, Galí, and Gertler 1999).

> "A permanent rise in productivity raises potential output, but it also raises output demand in a perfectly offsetting manner, due to the impact of permanent income. As a consequence, the output gap does not change. In turn, there is no change in inflation. Thus, there is no reason to raise interest rates, despite the rise in output…This kind of scenario seems to describe well the current behaviour of monetary policy [in the U.S. economy]. Output growth was substantially above trend in recent times but with no apparent accompanying inflation." (Clarida, Galí, and Gertler 1999. Term in brackets added.)

Apparently, the authors assume that increases in potential output are "perfectly" observed by individuals and interpreted as leading to an equivalent rise in expected lifetime wealth. In turn, insofar as individuals tend to smooth consumption overtime, the expectation of higher expected lifetime wealth leads them to consume more, both in the present and in the future. Formally, this assumption amounts to saying that the natural interest rate is *not* affected by changes in the "natural" rate of growth or in potential output. However, by resorting to this mechanism, proponents of the NCV tend to assume the problem away. Even if we accept that consumption is determined by individuals' wealth, there is still the problem that the level of potential output cannot be observed in practice.

The aggregate demand function presented above also implies that the central bank (hereafter CB) offsets stochastic shocks to aggregate spending by manipulating real interest rates. This is a fair enough assumption. In this respect, we believe that the most salient feature of the institutional framework that characterizes most, if not all, present-day OECD economies is that the CB fine tunes the economy through changes in short-term nominal interest rates, which ultimately affect the level of aggregate demand through a number of channels. That said, short-term nominal interest rates are subject to a zero lower-bound (ZB hereafter). This constraint arises because, in a money using economy, individuals will not be willing to hold any financial asset other than money when the nominal yield of the former is equal or less than zero. This institutional feature was termed by Kaldor (1939) the "great constitutional weakness" of monetary policy because it prevents the short-term nominal interest rate from operating equally freely in both directions. If we accept the proposition that CBs can and do affect aggregate demand by inducing pro-cyclical changes in real interest rates subject to the ZB constraint, then any demonstration of the possibility of the existence of an aggregate demand deficiency problem must adopt this institutional setting as the point of departure.

A core assumption of this study is that the adjustment of aggregate demand to potential output in the long run occurs mainly through the impact on aggregate demand of investment spending, as well as of conventional monetary policy actions. We define the latter as the regular actions that characterize the day-to-day setting of short-term nominal interest rates by CBs with a view to achieving an inflation target. Contrary to the NCV, we do not believe that permanent shocks to potential output *automatically* generate proportional increases in output so that in the absence of unusually large shocks CBs will always be in a position to generate the right level of aggregate demand. Rather, we believe that "structural" factors like the saving ratio or the "natural" rate of growth, as well as (transitory) demand and supply shocks, may be relevant to explain the emergence of an aggregate demand deficiency.

The main purpose of this study is to integrate in a simple framework the notion of the natural or neutral interest rate, liquidity preference theory (hereafter LPT),[3] and the stabilizing behavior of modern CBs in order to determine the circumstances under which an economy will experience an aggregate demand deficiency problem, i.e., a situation where the CB cannot generate a level of aggregate demand that is compatible with price stability through the manipulation of interest rates. Central to our theoretical framework is the notion of the natural or neutral interest rate (Wicksell 1936[1898]; Cassel 1928), which we define as the real interest on loans which is *neutral* with respect to the inflation rate and tends neither to raise nor to lower it in the absence of supply shocks. We wish to note that by utilizing the notion of the natural interest rate we do not mean that there is a tendency for the interest rate to converge to its natural level, as envisaged in loanable funds theory. In any case, we keep the term "natural" as it is standard use in the literature.

To construct this framework, we develop a model of a closed economy without a government sector in which a CB sets (real) interest rates with a view to hitting an inflation target. The key element in the analysis is the notion of the lower-bound trap (hereafter LBT) on real interest rates, which we define as a scenario in which the CB is

[3] Some examples of modern presentations of the notion of LP are the works by Wells (1983), Mott (1985–86), and especially, Lavoie (1996).

unable to push down real interest rates low enough so as to generate either a level of aggregate demand equal to potential output or a rate of growth of aggregate spending that is equal to the "natural" rate of growth.

According to us, the main contributions of this study are the following. First and foremost, we provide a simple theoretical framework that integrates the notion of the natural or neutral interest rate, LPT, and the praxis of monetary policy by modern CBs which allows (or so we believe) a richer and more realistic analysis of the circumstances under which an economy will exhibit an aggregate demand deficiency problem. Second, and contrary to the proponents of the NCV, we show that a combination of "structural" factors, like a high saving rate and a low "natural" rate of growth, as well as short-term or transitory shocks, may potentially make an economy experience an aggregate demand deficiency problem. Third, an implication of the former result is that we find support for those scholars who claim that the Japanese economy has been in a liquidity trap for a decade (Krugman 1998; Svensson 2005). In particular, we find support for those authors who point to the Japanese high saving rate and low "natural" rate of growth as the most likely causes of the recent Japanese stagnation. Fourth, we claim that the NCV model can only explain the existence of a liquidity trap by resorting to the occurrence of unusually large shocks. To be sure, the NCV aggregate demand function requires that current output is always equal to potential output whenever: 1) $g = 0$ and 2) r is equal to the steady growth interest rate. But, as we argue below, the latter will be the case if the economy is not in a LBT and, in turn, this will only occur if the sum of the natural interest rate and the inflation rate exceeds the term premium. We show that this will be the less likely the higher the saving rate and the lower the "natural" rate of growth. Fifth, contrary to conventional wisdom, we find that a rise (fall) in the NAIRU may lead to a fall (rise) in the natural interest rate.

The structure of the paper is as follows. Section 2 presents a simple model for a closed economy without a government sector. The steady growth properties of the model are worked out and discussed. We define formally the LBT and determine the size of the inflation (or supply) and demand shocks that will push our economy into a LBT. Section 3 contains the results of a simulation exercise that aims to determine the conditions under which an economy will get stuck in a LBT. Section 4 concludes.

2 THE MODEL

The equilibrium condition in the product market for a closed economy without a government sector when current output equals potential output is:

$$s(r)\cdot\overline{Y} = I \tag{1}$$

where *s* is the saving rate, $\overline{Y}$ is potential output, and *I* is the (gross) level of investment. The real interest rate that results from (1) is the Wicksellian or natural interest rate r^n, i.e., the real interest rate that makes planned saving at potential output equal to planned investment. We believe it is better thought of as a long-term interest rate. If we denote by $\overline{Y}$ the level of output that keeps inflation constant at a given period in the absence of transitory inflation or supply shocks (hereafter INSs), we have that inflation will rise (fall) when $r \prec r^n$ ($r \succ r^n$). This is the essential insight of the neo-Wicksellian approach to monetary policy (for a comprehensive presentation of this approach see Woodford 2003).

In addition to investment expenditure in productive capacity, we believe the most important regulation mechanism at work in modern market economies comes in the form of a CB that brings about changes in real interest rates so as to achieve an inflation target. We know that CBs can only set short-term nominal interest rates. Nevertheless, the presence of some inertia in the rate of inflation allows them to move real interest rates in the desired direction. The fact that the level of aggregate demand is a negative function of the real interest rate implies that CBs can, if nominal rates are above the ZB, set real interest rates so as to generate a level of aggregate demand that matches a given level of potential output. Let us present the following simple model:

$$\dot{\pi} = f_1(r - r^n) \tag{2}$$

$$r - r^n = f_2(\pi - \pi^d) \tag{3}$$

and substituting (3) into (2) yields:

$$\dot{\pi} = f_3(\pi - \pi^d) \tag{4}$$

where $f_1' \prec 0$, $f_2' \succ 0$, $f_3' \prec 0$, π is the inflation rate, π^d is the inflation target, and *r* is the current real interest rate. Differential equation (2) captures the dynamics of inflation in a neo-Wicksellian fashion. Equation (3) is a Taylor-type monetary policy rule. It can be seen that—since $f_3' \prec 0$—the rate of inflation will converge to π^d in (4). Thus, as long as the policy rule is governed by (3), the CB will hit π^d. Nevertheless, CBs face several difficulties when implementing a rule like (3). Of critical importance to our study is that there may be circumstances when nominal short-term interest rates may already be at the ZB so the CB will be unable to vary real interest rates as dictated by expression (3). If so, the economy will exhibit an aggregate demand deficiency problem. This situation is usually referred to as a liquidity trap.[4]

Next, let's consider a one-sector economy with two inputs, labor and capital, and assume that the aggregate production function has fixed coefficients. Potential output is defined as:

$$\overline{Y} = \lambda \cdot \overline{N} \leq v \cdot K \tag{5}$$

where $\overline{N}$ is the level of employment that keeps inflation constant in the absence of INSs and λ and *v* are, respectively, the productivity of labor and capital when the factors are fully utilized. The current rate of capacity utilization is:

$$u = \frac{Y}{v \cdot K} \leq 1 \tag{6}$$

Thus, the rate of capacity utilization when $Y = \overline{Y}$ is:

$$\overline{u} = \frac{\overline{Y}}{vK} = \frac{\lambda}{v} \cdot \frac{\overline{N}}{K} \prec 1 \tag{7}$$

where we define $\overline{u}$ as the constant inflation rate of capacity utilization or CICU (see Corrado and Mattey 1997). We denote by $\overline{e}$ the employment ratio corresponding to the constant inflation employment ratio or CIER and by *L* the labor force. For the sake of simplicity, we assume that the CIER is constant.[5] Hence, we have that expression (7) can be reexpressed as:

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[4] A discussion of this problem in the context of a static model capturing the basic features of the NCV in macroeconomics is in Arestis and Sawyer (2003). For instance, they point out that "if the Central Bank could vary the rate of interest to ensure that savings and investment were continuously equated at levels that corresponded to a supply-side equilibrium (perhaps full employment), then there would be no deficient demand problem" (Arestis and Sawyer 2003). They identify a number of reasons why this may not be the case.

[5] In turn, the non-accelerating inflation rate of unemployment or NAIRU is equal to $1 - \overline{e}$. The assumption that the CIER is constant and supply-side determined is a necessary compromise given the desirability of restricting the complexity of the model.

$$\bar{u} = \left(\frac{\bar{e}}{v}\right) \cdot \left(\frac{\lambda \cdot L}{K}\right) \prec 1 \tag{8}$$

where $\bar{N} = \bar{e} \cdot L$. In turn, the dynamics of the rate of inflation are given by:[6]

$$\dot{\pi} = \phi(u - \bar{u}) \qquad \phi \succ 0 \tag{9}$$

If we divide (1) through by the capital stock *K* and denote the rate of capital accumulation by *g* and the rate of depreciation of physical capital by ψ, we get:

$$s \cdot \frac{\bar{Y}}{K} = g + \psi \tag{10}$$

and inserting (7) into (10) yields:

$$s \cdot v \cdot \bar{u} = g + \psi \tag{11}$$

The current real wage *w/p* is determined by firms' profit maximization objectives:

$$\frac{w}{p} = \frac{\lambda}{m} \tag{12}$$

where *w* is the money wage, *p* is the price level, and $m \succ 1$ is one plus the (average) markup. For the sake of simplicity, we also assume that the latter is constant. Finally, the "natural" rate of growth is:

$$g_n = l + a \tag{13}$$

where *l* and a are, respectively, the growth rate of labor force *L* and labor productivity λ.

We now turn our attention to functions *s* and *g*. We assume that the saving rate (*s*) is a function of the rate of inflation (π), the rate of growth of output ($\hat{y}$), the real interest rate (*r*), and a measure of exogenous shocks (ε_s) or:

$$s = s(\pi, \hat{y}, r, \varepsilon_s) \tag{14}$$

where we assume that $s_{\hat{y}} \succ 0$, $s_\pi \prec 0$, $s_r \succ 0$, and ε_s is a stochastic variable with zero mean. The positive sign of $s_{\hat{y}}$ is based on the life cycle hypothesis of saving. The latter establishes a positive relation between *s* and $\hat{y}$ in the short and the long run (Modigliani and Brumberg 1980; Modigliani 1986). The positive sign of s_r stems from the fact that,

[6] The assumed linearity and symmetry of the inflation dynamics equation is also a necessary compromise. The short-run output-inflation trade-off is likely to be non-linear, albeit the precise shape—concave versus convex—is a matter of controversy (see Filardo 1998).

although households are on average net lenders and substitution and income effects move in opposite directions for those individual households who are net lenders, wealth effects operate in the same direction as the substitution effect, thus making $s_r \leq 0$ an unlikely scenario. The sign of s_π requires some clarification. In a study by Pollin (1985), the author shows that the stability of the total outstanding debt ratio (S_t) of the U.S. economy's non-financial sectors has displayed essentially no trend throughout the post-World War II period. Using the formula $S_t = q_t(1+\hat{Y})/\hat{Y}$, derived in Gurley and Shaw (1957), where q_t is the marginal propensity of the aggregate non-financial sector to issue net new debt and $\hat{Y}$ is the rate of growth of nominal *GNP*, the author argues that the stability of S_t throughout the postwar period, and especially since the 1960s, has resulted from rising trends for $\hat{Y}$ and q_t, coupled with a declining trend for the rate of growth of real *GNP*. As a result, the ratio $(1+\hat{Y})/\hat{Y}$ has fallen correspondingly over this period and q_t has risen along with $\hat{Y}$ in order for S_t to remain constant. According to Pollin (1985), the divergent patterns of S_t and q_t are due to the asymmetric impact of inflation on the two ratios. As for S_t, its numerator, the stock of debt, remains fixed in nominal terms regardless of variations in the price level (relative to trend), whereas its denominator, nominal *GNP*, varies in nominal terms directly with the price level. As a result, in an inflationary environment, the nominal value of the debt stock remains fixed while *GNP* rises, so that S_t is biased downwards. Conversely, with q_t, current period flow values are in both numerator and denominator, and thus, the impact of inflation on the ratio is neutral. Because of this asymmetry, an increasing reliance on debt by the non-financial sectors, i.e., a rising q_t, may not engender increases in their debt burdens.

Next, we have that for a given q_t, a fall in the rate of inflation will increase net borrowers' real debt burden and vice versa. In turn, this will increase the general level of bankruptcy risk. In the case of net borrower households, the increase in the real level of indebtedness will lead to a fall in consumption (Bernanke 1981). In the meantime, the increase in net borrowers' real debt burden will be coupled by a rise in net lenders' real financial wealth and, by the same token, this will tend to increase their consumption.

However, it is reasonable to assume that net borrowers' marginal propensity to consume out of wealth is, on average, higher than net lenders'. In addition, the possibility of bankruptcy has an asymmetric impact on aggregate spending. This is because, as the level of bankruptcy risk rises (owing to rising real debt burdens by net borrowers), the level of spending of net lenders rises by less than the fall in the level of spending by net borrowers. Hence, we have that $s_\pi \prec 0$. The size of s_π will be proportional to the size of the aggregate debt ratio and the degree of dispersion of balance sheet positions across households.

Next, let's assume that firms have a desired rate of capacity utilization $u^d \prec 1$, so they will expand capacity when $u \succ u^d$ and stop expanding it when $u \prec u^d$. A possible justification for this assumption is that firms prefer to keep some capacity idle in order to respond rapidly to unanticipated favorable demand shocks, as well as to deter the entry of potential rivals into the industry (Skott 1989). This assumption is equivalent to defining the rate of accumulation, (g) as:

$$g = v \cdot u \cdot f(u - u^d, \varepsilon_g) \tag{15}$$

where $f_u \succ 0$ is inversely proportional to the size of the lags affecting the construction and delivery of capital equipment, $f(0) = \bar{f} \succ 0$, $E(\varepsilon_g) = 0$, and ε_g represents stochastic shocks affecting g. Parameter $\bar{f}$ plays an important role in this model. Formally, it is the ratio of the level of net investment to output when $u = u^d$. More generally, it captures firms' average expected future rate of growth of demand. Therefore, expression (11) can be rewritten as:

$$s(\pi, \hat{y}, r, \varepsilon_s) = f(\bar{u} - u^d, \varepsilon_g) + \frac{\psi}{v\bar{u}} \tag{16}$$

Equation (16) represents the equilibrium condition in the product market when $Y = \bar{Y}$. The value of r that fulfills this equilibrium condition is the natural interest rate.

2.1 Steady Growth Analysis

In steady growth, we have that $\hat{y} = g_n$, $u = \bar{u} = u^d$, $\pi = \pi^d$, and $\varepsilon_s = \varepsilon_g = 0$, so that the two following conditions must hold:

$$v \cdot u^d \cdot \bar{f} = g_n \tag{17}$$

and
$$s(\pi^d, g_n, r^*) \cdot v \cdot u^d = g_n + \psi \tag{18}$$

Equation (17) tells us that in steady growth the rate of accumulation must equal the "natural" rate of growth. Equation (18) is the counterpart to equation (16) for the steady growth case. In order to get an explicit solution for the steady growth real interest rate (r^*), we assume that function s adopts a linear form or:

$$s = \bar{s} + s_{\hat{y}} \cdot \hat{y} + s_\pi \cdot \pi + s_r \cdot r \tag{19}$$

where $\bar{s}$ is a shift term determined by individuals' preferences and institutional factors. Substituting (19) into (18) and rearranging we get:

$$r^* = \left[\frac{g_n + \psi}{v \cdot u^d} - \bar{s} - s_\pi \pi^d - s_{\hat{y}} g_n \right] \frac{1}{s_r} \tag{20}$$

Thus, the steady growth real interest rate r^* is a function of the "natural" rate of growth, the target rate of inflation, the saving rate, the parameters of the saving function, the rate of depreciation, the technical capital output ratio, and the desired rate of capacity utilization. This result clearly runs against conventional wisdom since the latter poses that real variables are not affected by changes in nominal variables like the rate of inflation. Therefore, the model exhibits the property of nonneutrality of monetary policy in the long run. Finally, the steady growth real interest rate can also be interpreted as the real interest rate that is compatible with a neutral monetary policy in the long run. Hence, it represents a benchmark for the setting of interest rates for stabilization purposes. The steady growth properties of the model are:

$$\frac{\partial r^*}{\partial \pi^d} = \frac{-s_\pi}{s_r} \succ 0 \tag{21}$$

$$\frac{\partial r^*}{\partial g_n} = \left(\frac{1}{v \cdot u^d} - s_{\hat{y}} \right) \cdot \frac{1}{s_r} \succ 0 \tag{22}$$

$$\frac{\partial r^*}{\partial \bar{s}} = \frac{-1}{s_r} \prec 0 \tag{23}$$

The positive sign of (21) is the result of the expansionary effect on the level of aggregate demand of an increase in the rate of inflation. In principle, the sign of (22) is ambiguous. However, it will be positive for reasonable values of the parameters. This can be seen in Table 1, where we have that $\partial r^* / \partial g_n = 1{,}0\widehat{7}$ for the combination of parameters derived from the calibration of the model. This means that a 1 percent fall in the "natural" rate of growth leads approximately to a 1 percent fall in r^* and vice versa. Table 1 also shows that $\partial r^* / \partial \pi^d = 0{,}0\widehat{6}$, so we have that a 1 percent fall in the target rate of inflation leads to a 0.06 percent fall in r^*. Admittedly, this is a negligible effect. However, the simulation exercise we performed showed that reducing s_π tends to increase the stability of the system by making cyclical fluctuations more damped, so the actual value of s_π is subject to some uncertainty. For instance, if we set $s_\pi = -0.5$, we have that $\partial r^* / \partial \pi^d = 0.\widehat{3}$, so a 1 percent fall in the inflation target leads to a 0.3 percent fall in r^*. This suggests that changes in the rate of inflation probably have a small, albeit significant, impact on r^*. The negative sign of (23) reflects the fact that an increase in the saving rate reduces r^*. Finally, as the quotation above suggests, the NCV model assumes that $-\partial r^* / \partial \bar{s} = \partial r^* / \partial g_n$ if $d\bar{s} = d\, g_n$—but only when causality runs from g_n to $\bar{s}$—and $\partial r^* / \partial \pi^d = 0$.

Expression (22) also suggests that phenomena like the "New Economy," whereby g_n has allegedly increased due to the spread of information technology across the economy, has translated into a rise in r^*. As we argue below, this makes an economy less prone to getting stuck in a LBT and may also raise the CIER. More important to our discussion, it has been argued that the main cause of the stagnation experienced by the Japanese economy in the last decade is its high saving rate and low "natural" rate of growth (Krugman 1998; Nakatani and Skott 2006).[7] As expression (22) reflects, this hypothesis can be easily accommodated in our model so our results lend support to the views of those authors who argue that the Japanese economy was in a liquidity trap. This is because the low Japanese "natural" rate of growth, its high saving rate, and its low rate of inflation combine to yield a low and possibly negative steady growth interest rate. In a

[7] According to these authors, the low Japanese "natural" rate of growth is the result of a negative rate of growth of labor force due to adverse demographic trends and a low rate of growth of productivity owing to the completion of the technological catch-up phase. Nakatani and Skott (2006) argue that the low "natural" growth rate generates a low profit share in national income and they refer to this situation as a "structural" liquidity trap.

very low inflation environment, the former is likely to make the ZB on nominal rates binding.[8]

2.2 The Behavior of the Economy in the Short Run

Steady growth analysis is only valid for a long-run scenario in which the effects of shocks have already worked themselves out. The steady growth case also provides a position around which the economy will fluctuate. We now focus on the behavior of the economy in the short run. As we did with the saving rate, we assume that the investment function f adopts a linear form or:

$$f(u-u^d,\varepsilon_g)=\bar{f}+f_u\cdot(u-u^d) \qquad (24)$$

where f is a shift term capturing expectations on future demand growth. Substituting (24) into expression (16), we obtain the short-run equilibrium condition in the product market:

$$\bar{s}+s_{\hat{y}}\cdot\hat{y}+s_\pi\cdot\pi+s_r\cdot r=\bar{f}+f_u\cdot u-f_u\cdot u^d+\frac{\psi}{v\cdot u} \qquad (25)$$

and, as a result, the short-run rate of growth of output, $\hat{y}$ is:

$$\hat{y}=\frac{\bar{f}-\bar{s}+f_u\cdot u-f_u\cdot u^d-s_\pi\cdot\pi-s_r\cdot r}{s_{\hat{y}}}+\frac{\psi}{s_{\hat{y}}\cdot v\cdot u} \qquad (26)$$

so that:

$$\frac{\partial\hat{y}}{\partial r}=\frac{-s_r}{s_{\hat{y}}}\prec 0 \qquad (27)$$

and

$$\frac{\partial\hat{y}}{\partial\pi}=\frac{-s_\pi}{s_{\hat{y}}}\succ 0 \qquad (28)$$

Hence, an increase in r will raise $\hat{y}$, whereas an increase in π will reduce it. Rearranging (25), we obtain the expression for the rate of interest that clears the product market in the short-run when $\hat{y}=0$ and $\hat{y}=g_n$, respectively, or:

$$r^{\hat{y}=0}=\frac{\bar{f}+f_u\cdot u-f_u\cdot u^d-s_\pi\cdot\pi-\bar{s}}{s_r}+\frac{\psi}{s_r\cdot v\cdot u} \qquad (29)$$

[8] The competing explanation to the LBT in Japan is the “credit crunch” view. This explanation focuses on the contraction of the supply of credit (credit crunch) by Japanese banks in the 1990s caused by massive nonperforming loans accumulating in the financial system (Cargill, Hutchison, and Ito 1997). However, we believe that this explanation can be incorporated in our model. As argued above, if banks restrain the flow of credit to the non-bank private sector, then aggregate demand and the natural interest rate will fall.

$$r^g = \frac{\bar{f} + f_u \cdot u - f_u \cdot u^d - s_\pi \cdot \pi - \bar{s} - s_{\hat{y}} \cdot g_n}{s_r} + \frac{\psi}{s_r \cdot v \cdot u} \quad (30)$$

Expression (29) corresponds to point *A* in Figure 1 below. In expression (30), r^g is the rate of interest that yields $\hat{y} = g_n$ for a given *u* and π. Therefore, the difference between r^g and r^* is that in the former it will generally not be the case that $\pi = \pi^d$ and $u = u^d$. Setting $r = 0$ in (26), we obtain point *B* in Figure 1 or:

$$\hat{y}^{r=0} = \frac{\bar{f} - \bar{s} + f_u \cdot u - s_\pi \cdot \pi - f_u \cdot u^d}{s_{\hat{y}}} + \frac{\psi}{s_{\hat{y}} \cdot v \cdot u} \quad (31)$$

Before we look at Figure 1, we need to analyze the impact of INSs and demand shocks (hereafter DSs) on r^g. As expression (32) below shows, unfavorable (favorable) INSs lead to a rise (fall) in r^g. In turn, expressions (33) and (34) show that favorable (unfavorable) DSs lead to a rise (fall) in r^g.

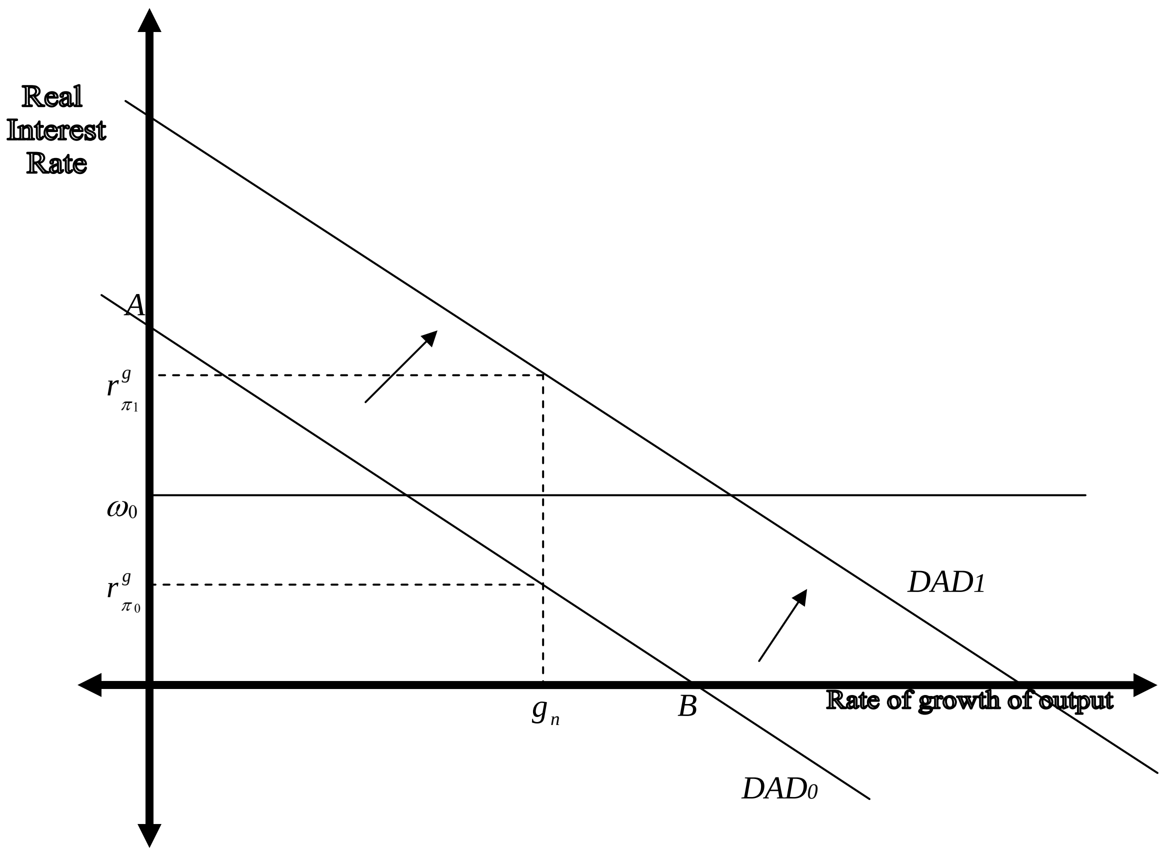
$$\frac{\partial r^g}{\partial \pi} = \frac{-s_\pi}{s_r} \succ 0 \quad (32)$$

$$\frac{\partial r^g}{\partial \bar{f}} = \frac{1}{s_r} \succ 0 \quad (33)$$

$$\frac{\partial r^g}{\partial \bar{s}} = \frac{-1}{s_r} \prec 0 \quad (34)$$

The determination of r^g is illustrated graphically in Figure 1. We measure the real interest rate (*r*) in the vertical axis and the rate of growth of output $\hat{y}$ in the horizontal one. The line denoting $\hat{y}$ for every value of *r* is downward-sloping, as stems from (27). We refer to it as the dynamic aggregate demand line or *DAD* line. In general, it will not be a line, owing to the presence of *u* and π in expression (26). We impose on it the assumption of linearity for the sake of the argument. Its position is determined by the current rate of inflation and capacity utilization. Since $\partial r^g / \partial \pi \succ 0$, we have that an increase in inflation, from say π_0 to π_1, shifts the *DAD* line upwards from DAD_0 to DAD_1, thereby leading to a rise in r^g. Similarly, a rise in $\bar{f}$ or a fall in $\bar{s}$ will shift the *DAD* line upwards. We will return to Figure 1 after defining the LBT.

Fig. 1: The determination of r^g



Next, we define the natural or neutral interest rate (r^n) as the real interest rate that yields a level of aggregate spending such that $u = \bar{u}$. It is clear from expression (35) below that r^n is a function of π, $\hat{y}$, $\bar{f}$, and $\bar{s}$. An explicit solution of r^n can be obtained from (25) by setting $u = \bar{u}$ and rearranging:

$$r^n = \frac{\bar{f} + f_u \cdot \bar{u} - f_u \cdot u^d + \dfrac{\psi}{v \cdot \bar{u}} - \bar{s} - s_{\hat{y}} \cdot \hat{y} - s_\pi \cdot \pi}{s_r} \qquad (35)$$

As expression (35) reflects, r^n depends on a wide range of factors, including the parameters of the saving and investment function, the rate of growth of output, capacity utilization, the inflation rate, and, importantly, the CICU. It is easy to see that an increase in $\bar{s}$ or a fall in $\bar{f}$ associated with an increase in LP will push r^n downwards. By contrast, a rise in π will push it upwards. At first sight, it may seem as if this interpretation of liquidity preference (hereafter LP) leaves aside some insights of modern presentations of LPT, for example, the role played by banks in setting quantity constraints to credit (Moore 1988; Heise 1992; Wray 1992; Lavoie 1996; Wolfson 1996). Post Keynesian scholars have usually identified the concept of LP with an environment of fundamental uncertainty, where LP is inversely related to the degree of confidence, the reliability of beliefs, and Keynes' animal spirits. For instance, an increase (fall) in the LP of banks will make them restrain (expand) the creation of credit money (and possibly to raise the loan rate *vis á vis* the cost of obtaining funds in wholesale money markets) which, in turn, will reduce aggregate demand. The crucial point for our argument is that the impact upon aggregate demand of this type of action will be akin to the effect in our model of an increase in $\bar{s}$ or a fall in $\bar{f}$, in that it will push r^n downwards.

Equation (35) also highlights that a change in the CICU will have an uncertain impact on r^n. This is because, for instance, a rise in the CICU leads—via equation (11)—to a rise in the rate of accumulation that may or may not be fully offset by an increase in the flow of saving at potential output.[9] We would like to make three considerations about this result. First, the former means that the potential presence of hysteresis in the CIER will have an uncertain impact on the likelihood that the economy is subject to the LBT constraint. This is because, in the presence of hysteresis, as the employment ratio falls, the CIER will follow suit, but this, in turn, may push r^n downwards or upwards. Second, it has been argued that the rise in productivity growth that occurred in the U.S. economy in the 1990s led to an increase in the CIER—or a fall in the NAIRU—owing to the slow adjustment process of workers' wage aspirations to the higher productivity growth exhibited by the U.S. economy in that period (see Ball and Mankiw 2002). This process might have operated in the opposite direction in Japan, so it is possible that as Japanese productivity growth slowed down in the 1990s, its CIER and hence, its CICU, fell, thus leading to a fall in r^n. This story is consistent with the Japanese stagnation and the stationary tendencies exhibited by the Japanese inflation rate

[9] More precisely, we have that the sign of $d\, r^n / d\bar{u} = 1/s_r \cdot \left(f_u - \dfrac{\psi}{v\,\bar{u}^2} \right)$ is ambiguous.

in the same period despite rising registered (and hidden) unemployment.[10] In the case of the United States, it may be that a rising CIER led to a higher r^n. As argued in Ball and Tchaidze (2002), the failure of the Fed to raise the federal funds rate in response to falling unemployment is explained by a decrease in the NAIRU in the late 1990s. As they show (see their figure 1 in particular), the stationary federal funds rate, coupled with a slight downward trend of the inflation rate in that period, resulted in the real federal funds rate exhibiting an *upward* trend which is consistent with a higher r^n. Be that as it may, and this is our third consideration, these phenomena cannot be explained by the NCV model which predicts that changes in the "natural" rate of growth or in potential output affect neither the CIER nor the natural interest rate. This is because, as implied by the quotation of Clarida, Galí, and Gertler (1999) in the introduction, a rise in potential output raises output demand in a perfectly offsetting manner due to the impact of permanent income which, in turn, implies that changes in the CIER do not affect the natural interest rate. In any case, conventional wisdom has it that a rise in the CIER—or a fall in the NAIRU—will be associated with a fall rather than a rise in the natural interest rate and vice versa, since a higher CIER represents a larger flow of aggregate saving at (higher) potential output that, in principle, is not compensated by higher investment demand.

2.3 The Lower-bound Trap (LBT)

We now seek to provide a formal and general definition of a LBT on real interest rates. We distinguish between two different types of LBT: the growth lower-bound trap (GLBT) and the conventional lower-bound trap (CLBT).

[10] As shown in Kuttner and Posen (2001), with the exception of a brief respite in 1997 (in large part due to an increase in the consumption tax) inflation, as measured by the Japanese GDP deflator, was consistently negative between 1994 and 2001, but failed to exhibit a clear downward trend. One reason for this behavior of the Japanese inflation rate is provided in Yellen and Akerlof (2004) who argue that the evidence suggests a diminished tendency for inflation to fall in the face of high unemployment due to a lower pass-through of inflationary expectations into inflation once inflation is already low. They also blame downward nominal wage rigidity for this effect. In this respect, Mourougane and Ibaragi (2004) estimate Phillips curves for Japan and find strong evidence that at low or negative inflation rates, indicators of demand pressure have no statistically significant effect on price inflation. This implies that a more realistic presentation of equation (9) above would be to set a lower value for its parameter when u is below the CICU than when it is above.

2.3.1 The Growth Lower-bound Trap (GLBT)

If we denote by ω the *minimum* (ex-ante) real interest rate that a CB can set, we can define a GLBT on real interest rates as a situation where:

$$r^g \prec \omega \tag{36}$$

If we think of *r* as a short-term real interest rate, then it is reasonable to assume that the minimum nominal interest rate the CB can set is zero. However, if we think of *r* as a long-term interest rate, then the minimum nominal interest rate the CB can set will be positive. This is because investors require a (time-varying) term premium, $\mu \succ 0$, to purchase long-dated securities.[11] We believe that the size of μ is mainly determined by the LP of investors. If we also assume that the expected rate of inflation is equal to the current inflation rate we have:

$$\omega = \mu - \pi^e = \mu - \pi \tag{37}$$

Therefore, we will say that an economy is in a GLBT if:

$$r^g + \pi \prec \mu \tag{38}$$

This corresponds to the case illustrated in Figure 1 above. The horizontal line denotes the value of ω for a rate of inflation π_0. Hence, the economy will be in a GLBT insofar as $r^g \prec \mu - \pi_0 = \omega$. As a result, we have that $\hat{y} \prec g_n$ and either an initially positive output gap will narrow or a negative one will widen. Likewise, if the economy is in a GLBT, we will observe that the unemployment rate exhibits an upward trend despite short-term nominal interest rates being stuck at zero.[12] This scenario fits well in Japan's recent experience since, as shown in Orphanides (2004), its rate of unemployment rose steadily from about 2 percent in 1990 to more than 5 percent in 2002. But this increase in

[11] We may note that the nominal rate of return on long-dated securities is determined by an average of the current and expected future short-term nominal interest rates plus the term premium required by investors—which will be positive—so that, in principle, the difference between the nominal return of long-dated and short-dated securities will have an uncertain sign. However, when short-term nominal interest rates are already at zero, the yield curve cannot be downward-sloping because expected future short-term nominal interest rates will be at or above zero, so the difference between long-term and short-term real interest rates is well approximated by the term premium required by investors.

[12] If an economy has been in a GLBT for a relatively long period, one would expect that it exhibits a large (negative) output gap. In this respect, Krugman (1998) argues that the negative output gap the Japanese economy exhibited in the late 1990s was largely underestimated in official statistics. He adds that it could have been as large as 8 per cent of GDP and it may have grown much larger since 1998. Kuttner and Posen (2001) make a similar point and provide estimates of the Japanese output gap.

registered unemployment does not tell us the whole story, since hidden unemployment was also on the rise in Japan (see Nakatani and Skott 2006).

Next, a favorable INS will push the economy into a GLBT if:

$$d\pi \prec \frac{-(r_0^g + \pi_0 - \mu)}{1 + \partial r^g / \partial \pi} \tag{39}$$

where the subscript in π and r^g denotes the initial values of these variables. Finally, an unfavorable DS due, for instance, to a rise in $\bar{s}$, will push the economy into a GLBT if:

$$\pi_0 + r_0^g - \mu + \frac{\partial r^g}{\partial \bar{s}} \prec 0 \tag{40}$$

An economy that finds itself in a GLBT may not necessarily be stuck in a CLBT. In fact, if the employment ratio is decreasing (increasing) but is still above (below) the CIER, the rate of inflation will keep rising (falling), at least for some time. Therefore, the notion of the GLBT allows us to understand the circumstances under which the CB will be unable to prevent the employment ratio from falling and which type of shock may pull the economy out of that scenario. We now look at the circumstances under which the CB will not be able to prevent the inflation rate from falling in the absence of adverse INSs.

2.3.2 The Conventional Lower-bound Trap (CLBT)

We define a CLBT as a situation in which conventional monetary policies have become impotent because nominal interest rates are at or near zero. In turn, this will be the case when desired saving exceeds desired investment at the CIER, even at a zero short-term nominal interest rate or $r^n \prec \omega$. If an economy gets stuck in a CLBT, the rate of capacity utilization will be lower than the CICU and inflation will fall steadily in the absence of adverse INSs.[13] A CLBT represents a situation of involuntary unemployment in the sense that money wage cuts will not help the economy reduce the unemployment rate. Indeed, in these circumstances, money wage cuts and, more generally, downward price flexibility, may make matters even worse by setting off a deflationary spiral. This is because once the short-term nominal interest rate is zero, additional reductions in the rate

[13] Of course, even if $r^n \succ \omega$, there is still the possibility that a CB sets real interest rates above the natural interest rate in order to generate a disinflation process.

of inflation will push up real interest rates further reducing aggregate demand. Assuming that expected inflation equals current inflation, we say that an economy is in a CLBT if:

$$r^n + \pi \prec \mu \tag{41}$$

Expression (41) tells us that the lower r^n and π are, the more likely it is that the economy gets stuck in a CLBT in the wake of favorable INSs or unfavorable DSs. Thus, setting an inflation target that is close to zero increases the probability that the economy gets in a CLBT. In contrast to recent discussions in the literature, expressions (38) and (41) also highlight that a negative r^g or r^n—due, for instance, to a high saving rate or pessimistic expectations about future demand growth—or a high μ are as potentially dangerous as the setting of a very low inflation target.[14]

Conventional wisdom regarding the ZB trap on nominal interest rates seems to be that the latter may arise as a result of unusually large adverse DSs or favorable INSs in a very low inflation environment. It also holds that this is a very unlikely scenario, as long as target (measured) inflation is at or above 2 percent (Fuhrer and Madigan 1997; De Long 1999). However, the literature tends to sidestep the fact that, as expressions (38) and (41) reflect, an economy may get into a LBT even if inflation is not very low, i.e., if r^n is low or negative and μ is high. In turn, a low or negative r^n will be the result of a high saving rate, a low "natural" rate of growth, or a combination of them.

Finally, expressions (38) and (41) also suggest that a possible escape route from a LBT is the creation of inflationary expectations, as proposed in Krugman (1998) and Eggertsson and Woodford (2003). However, as long as expected inflation tracks current inflation, there is no clear way through which a CB can overcome the so-called "inverted credibility" problem.[15] The experience of disinflation episodes of OECD economies in the last two decades suggests that inflation expectations closely track current inflation. As a result, CBs can only raise inflationary expectations by generating inflation and they

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[14] In the context of an open economy with a government sector, a low or negative natural interest rate may also be the result of either a large government surplus or a large current account deficit.

[15] Krugman (1998) argues that if a CB can credibly commit to pursue inflation and ratify inflation when it comes, it should be able to increase inflationary expectations despite the absence of any traction on the economy by means of conventional monetary policy. Eggertsson and Woodford (2003) present a more fully dynamic analysis of the problem. To them, a commitment to create subsequent inflation is presented as a commitment to keep interest rates low for some time in the future.

cannot generate it as long as the economy remains stuck in a LBT.[16] Therefore, in the absence of unconventional monetary policy options, only expansionary fiscal policy or substantial improvements in the current account balance will kick start the economy.[17]

We now turn to the analysis of the behavior of the natural interest rate in the aftermath of INSs and DSs and to a quantitative assessment of the likelihood that an economy be driven to a LBT in the aftermath of INSs and DSs. Differentiating (36) and rearranging we obtain:

$$d\,r^n = \frac{-s_\pi \cdot d\pi - s_{\hat{y}} \cdot d\hat{y} - d\bar{s} + d\bar{f}}{s_r} \tag{42}$$

If we initially set $d\hat{y} = d\bar{s} = d\bar{f} = 0$ then (42) becomes:

$$d\,r^n = \Pi_0 \cdot d\pi \tag{43}$$

where $\Pi_0 = \frac{-s_\pi}{s_r} \succ 0$.

[16] As pointed out by Blinder (2000), "the problem, in a word, is that such a policy pronouncement will not be credible once a country is already in the soup." The words of Kazuo Ueda at the 1999 JMCB Conference in Woodstock (Vermont) may serve to summarize the difficulties faced by a CB once the ZB on nominal rates binds: "Don't put yourself in the position of zero interest rates. You'll have to face a lot of difficulties. I can tell you it will be a lot more painful than you can possibly imagine" (Ueda 2000).

[17] A number of authors have recently analyzed the likelihood of economies getting stuck in a LBT, as well as the policy options that will remove this constraint in case the economy gets there. Two recent Journal Symposiums have been devoted to this issue; the 2000 Symposium on "Monetary Policy at the Zero Lower Bound" published in the *Journal of Money, Credit, and Banking* and the 2004 Symposium on "Policies to Deal with Deflation" in the *American Economic Review*. A summary of the former is in Blinder (2000) and an evaluation of the different proposals is in Bernanke and Reinhart (2004). There seems to be an emerging consensus on the view that the mere existence of a ZB on nominal interest rates can engender a moderate deterioration in macroeconomic stability as the inflation target approaches zero so that it represents an important constraint on how monetary policy operates in a low inflation environment (Fuhrer and Madigan 1997; Reifschneider and Williams 2000; Mussa 2000). In particular, studies find that there is trade-off between the average rate of inflation and the variability of the output gap so that the latter increases as the rate of inflation approaches zero and becomes negative (see Orphanides and Wieland 1998). One aspect these studies address is the possibility that the economy enters a deflationary spiral once the ZB binds. The usual verdict is that this type of episode is very rare and thus, it should not be a matter of concern. Yet, they recommend that CBs set a low, but positive, inflation target (preferably 2 per cent). Conversely, there is no emerging consensus as to whether unconventional monetary policy options can take the economy out of a CLBT should it be necessary. Several policy options for CBs to deal with it have been proposed (see Goodfriend 2000, 2001; Clouse *et al.* 2000; McCallum 2000; Svensson 2001; Yates 2002; Buiter 2003; Ito and Mishkin 2004; Bernanke, Reinhart, and Sack 2004). However, as emphasized in Reifschneider and Williams (2000) "the likely effectiveness of such actions is unclear from a theoretical perspective, and they have never been put to a definitive test." Only expansionary fiscal policy tends to be supported as the most reliable weapon in a liquidity trap (see, in particular, Kuttner and Posen 2001).

Thus, INSs *per se* induce changes of r^n in the same direction as current inflation, so r^n will move in a destabilizing fashion. As a result, any CB-induced change in real interest rates will be less effective in affecting output and inflation than if r^n remained constant. If we retrieve expression (41), we may conclude that, in the absence of changes in $\hat{y}$, the economy will get stuck into a CLBT in the wake of a favorable INS ($d\pi \prec 0$) if:

$$d\pi \prec \frac{-(r_0^n + \pi_0 - \mu)}{(1+\Pi_0)} \quad (44)$$

It is clear from (44) that, in the absence of changes in $\hat{y}$ and for given values of π_0, r_0^n, and μ, the likelihood that the economy gets stuck into a CLBT is proportional to the magnitude of the initial INS ($d\pi$) and the size of Π_0. If we use the values of the parameters obtained from the calibration of the model (see Table 1 below), assuming that $\pi_0 = 0.02$ and $\mu = 0.01$, and taking the value of r^* in Table 1 as a proxy for r_0^n, we have that $\Pi_0 = 0.06$ so the minimum size of a favorable INS that will push the economy into a CLBT is equal to –3.73 percentage points. Admittedly, such a favorable INS is extremely unlikely. However, an increase in $s_{\hat{y}}$, $\bar{s}$, or μ, or a fall in g_n will reduce this value significantly. For instance, if $g_n = 0.015$, we have that $r^* = 0.003$ and so the minimum size becomes as low as –1.2 percentage points.

In a closed economy without a government sector, DSs initially affect r^n through their direct impact on *s* or *g*. For the sake of convenience, we restrict the analysis to the case of shocks affecting $\bar{s}$ ($d\bar{s} \neq 0$). Notwithstanding, results are qualitatively similar for shocks affecting *g* ($d\bar{f} \neq 0$). If we set $d\pi = d\hat{y} = d\bar{f} = 0$ in (42) we obtain:

$$d\, r^n = \Pi_1 \cdot d\bar{s} \quad (45)$$

where $\Pi_1 = \dfrac{-1}{s_r} \prec 0$.

Expression (45) tells us that the impact on r^n of a shock to the saving rate when $d\pi = d\hat{y} = d\bar{f} = 0$ is proportional to Π_1. Since Π_1 is negative, we have that the behavior of r^n will also be destabilizing. If condition (41) does not hold, then the minimum size of the variation in the real interest rate (Δr) required to offset a shock to $\bar{s}$ is:

$$\Delta r = (r_0^n - r_0) + \Pi_1 \cdot d\bar{s} \quad (46)$$

where r_0 is the initial current real interest rate. In turn, the economy will get stuck into a CLBT in the wake of an adverse DS ($d\bar{s} \succ 0$) if:

$$d\bar{s} \succ \frac{-(r_0^n + \pi_0 - \mu)}{\Pi_1} \tag{47}$$

If we make use of the values of the parameters derived from the calibration of the model we have that, assuming that $\pi_0 = 0.02$ and $\mu = 0.01$ and taking the value of r^* as a proxy for r_0^n, we have that $\Pi_1 = -0.6$, so the minimum rise in $\bar{s}$ required to push the economy into a CLBT is equal to 0.066 and this value gets as low as 0.021 if $g_n = 0.015$. Given the high degree of uncertainty surrounding the values of the parameters, these results suggest, as a minimum, the need to complement monetary policy with counter-cyclical fiscal policy. To be sure, if the economy gets stuck in a LBT, then a sufficiently large increase in the government budget deficit will raise both r^n and r^g, thus allowing monetary policy to push the economy out of the LBT.

3 QUANTITATIVE ANALYSIS

Appendix B presents the details of a simulation exercise aimed at calibrating the model presented in Section 2 above. In that exercise, we assign plausible values to the parameters of the model so as to compute the multipliers and partial derivatives obtained in Section 2, as well as the operators used in Section 3. In this case, plausible values are a set of values that coincide with the values found in empirical studies when the latter are available or else a set of values that render the model stable. In order to evaluate the behavior of the economy in the presence of a ZB on nominal interest rates, we first need to specify the monetary policy rule. For the sake of the argument, we assume that the CB knows the steady growth interest rate (r^*) and that the (long-term) real interest rate is determined according to the following rule:[18]

[18] Although CBs can only set short-term interest rates, it is only a short step forward to assume that they also set long-term interest rates if the term premium and investors expectations on future short-term interest rates are constant.

$$r_t = \begin{cases} \omega_t = \mu - \pi_t & if \quad r^* + \alpha \cdot (\pi_t - \pi^d) \prec \omega_t \\ r^* + \alpha \cdot (\pi_t - \pi^d) & if \quad r^* + \alpha \cdot (\pi_t - \pi^d) \geq \omega_t \end{cases} \tag{48}$$

where α is the inflation gap response coefficient of the CB policy rule.

Therefore, the real interest rate is determined by a Taylor-type rule as long as the ZB does not bind or else is equal to the difference between the term premium and the inflation rate. Figures 2 through 5 below show the time path of the inflation rate, capacity utilization, the CICU, the current and natural interest rate, and the CLBT condition ($r^n + \pi - \mu \prec 0$). It can be seen that, for the set of parameter values reported in Table 1 below, the economy exhibits damped oscillations and converges to the singular point, P_1^* (see Appendix A for a derivation of the singular points of the dynamical system). It can be seen in Figure 5 that the CLBT condition exhibits cyclical oscillations with more amplitude than either the natural or the current interest rate. The time paths of the natural and current real interest rate are remarkably similar so they cannot be distinguished from each other in practice. This suggests that the high volatility exhibited by the natural interest rate makes it extremely difficult for CBs to use it in practice as a benchmark for setting real interest rates.[19]

Importantly, the simulation exercise revealed that increases in the saving rate or the term premium and/or reductions in the "natural" rate of growth or the inflation target increase the vulnerability of the system. In other words, they increase the likelihood that the economy gets stuck in a LBT. When this is the case, the economy collapses abruptly. This is due to the fact that when the economy hits a LBT and the output gap is negative, the rate of inflation keeps falling, thus pushing real interest rates further up. In turn, the rise in the real interest rate increases the size of the (negative) output gap further, thereby

[19] In addition, its volatility finds support in recent studies. For instance, in a study by Laubach and Williams (2001), the natural rate of interest is estimated using quarterly data for the United States over the period 1961 to 2000. In all the model specifications displayed, the authors find substantial variations in its estimated value throughout that period. For instance, in the baseline specification, the minimum value of the natural interest rate is found to be as low as 1.28 percent, whereas the maximum value is found to be 4.52 percent. In addition, the authors concede there is sizeable uncertainty around most of their estimates of the natural rate of interest, the trend growth rate, and potential output so the actual variation could be much larger. Crespo Cuaresma, Gnan, and Ritzberger-Grünwald (2005) provide a comprehensive survey of international natural rate of interest estimates.

setting off a deflationary spiral.[20] For instance, for the set of parameter values reported in Table 1, the economy fell apart or diverged gradually from the steady growth position if $g_n \prec 0.0138$. This value rises to 0.017 if $\mu = 0.015$. Similarly, the economy collapses if $\bar{s} \succ 0.142$, but this value falls to 0.135 if $\mu = 0.015$. Finally, the economy collapses if $\mu \succ 0.038$, but this value gets as low as 0.012 if $g_n = 0.015$. However, these results were obtained for the initial conditions reported in Table 1. They may change considerably if the initial conditions differ markedly from those reported. In general, the closer the initial conditions for u, $\bar{u}$, and π are to point P_1^*, the less likely it is that the economy gets stuck in a LBT for a given set of parameter values.

4 CONCLUSION

This study attempted to integrate in a simple theoretical framework the notion of the natural or neutral interest rate, liquidity preference theory, and monetary policy practice by modern CBs in order to analyze the conditions under which an economy will exhibit an aggregate demand deficiency problem. We argued that a CB will be able to prevent this scenario if the economy is not stuck in a lower-bound trap (LBT) and showed that this will be the case when the difference between the natural or neutral interest rate and the term premium required by investors is higher than the rate of inflation with a negative sign. Liquidity preference plays an important role in this framework since it affects the natural interest rate, as well as the term premium. We also argued that the predictions of the model broadly support the hypothesis that the Japanese economy has been in a LBT for almost a decade. This is because an implication of our analysis is that "structural" factors, like a high saving rate or a low "natural" rate of growth, increase the likelihood that a LBT on real interest rates binds. Our results stand in contrast to the New Consensus

[20] As recognized in Reifschneider and Williams (2000), this type of result holds for almost any macroeconomic model in which: 1) monetary policymakers influence aggregate demand primarily through changes in real short-term interest rates and 2) inflation displays significant inertia. To avoid a catastrophic collapse in (stochastic) simulation resulting from these deflationary episodes, they make allowance in the formation of expectations for the possibility of emergency fiscal stimulus in cases of persistent periods of zero rates. The stimulus is assumed to be of sufficient magnitude to offset the effect of the ZB until the economy recovers.

View model in macroeconomics which predicts that a LBT may only occur as a result of unusually large shocks in a very low inflation environment.

Although we did not address the mainstream literature on the policy options to take an economy out of a liquidity trap, our analysis suggests that counter-cyclical fiscal policy is still the most reliable tool. Another prediction of our model that runs against conventional wisdom is that an increase in the NAIRU may involve a fall, rather than a rise, in the natural interest rate and vice versa. We claimed that this prediction is consistent with the experiences of the United States and Japan in the 1990s. In any case, the implications of this result are far reaching and could not be discussed at length in this study. Finally, the most obvious limitation of our framework is the (assumed) exogeneity of the NAIRU and the "natural" rate of growth. Hence, a future development of this paper is to study the impact on our results of the interaction between the former two variables and the level of aggregate demand.

APPENDIX A: COMPUTATION OF SINGULAR POINTS

In order to compute the singular points of the model presented in Section 2, we need to rewrite expression (3) in linear form or:

$$r = r^{*} + \alpha(\pi - \pi^{d}) \tag{49}$$

where $\alpha \succ 0$ is the response coefficient of monetary policy to changes in the inflation gap; we assume for simplicity that the CB knows r^{*}. This policy rule implies that policymakers respond in a systematic and symmetric fashion to deviations of inflation from target. By definition, we have that:

$$\frac{\dot{u}}{u} = \hat{y} - g \tag{50}$$

and substituting (24) and (26) into (50) yields:

$$\dot{u} = h(u, \pi) \tag{51}$$

Next, if we take logarithms in (8) and differentiate it with respect to time we get:

$$\dot{\bar{u}} = \bar{u} \cdot (g_n - g) \tag{52}$$

and substituting (24) into (15) and then into (52) yields:

$$\dot{\bar{u}} = \bar{u} \cdot \left[g_n - v \cdot u \cdot (\bar{f} + f_u \cdot (u - u^{d}))\right] \tag{53}$$

Therefore, our dynamical system is made up of differential equations (9), (51), and (53). For convenience, we assume that $\bar{f} = (g_n / v \cdot u^{d})$, as stems from expression (17). We can then obtain the singular points of the system made up of equations (9), (51), and (53) by setting $\dot{\pi} = \dot{u} = \dot{\bar{u}} = 0$ which, in turn, yields the two following singular points:

$$u^{*}_{1,2} = \frac{\bar{f}v - v f_u u^{d} \pm \sqrt{(-\bar{f}v + v f_u u^{d})^{2} + 4v f_u g_n}}{-2v f_u}$$

and $$\pi^{*}_{1,2} = \frac{\bar{f} - \bar{s} + f_u(u - u^{d}) - s_r r^{*} + s_r \alpha \pi^{d} + \dfrac{\psi}{v u^{*}} - s_{\hat{y}} v \bar{f} u^{*} - s_{\hat{y}} v f_u u^{*2} + s_{\hat{y}} v f_u u^{d} u^{*}}{s_\pi + s_r \alpha}$$

where the only singular point with economic meaning is:[21]

$$P_1^{*} = (\pi^{d}, u^{d}, u^{d})$$

[21] The second singular point given the values assigned to the parameters and reported in Table 1 above is: $P_2^{*} = (-0.36; -2.5; -2.5)$.

APPENDIX B: NUMERICAL SIMULATION

The numerical simulation exercise was aimed at calibrating the model presented in Section 2. This exercise allowed us to compute the multipliers, operators, and partial derivatives obtained in previous sections. Table 1 reports the values of the operators computed for the formal analysis, as well as the values for r^*, $\bar{f}$, $\partial r^*/\partial g_n$, $\partial r^*/\partial \pi^d$, $\partial r^*/\partial \bar{s}$, $\partial r^*/\partial \psi$, and $\partial \Delta_4/\partial \alpha$. It also reports the value of α^*, i.e., the minimum value of the response coefficient of the monetary policy rule of the CB that makes the system stable. Figures 2 through 5 show the time path of all the variables in the dynamic model for the values of the parameters reported in Table 1. The parameter values were chosen according to the values reported in the empirical literature. For instance, the long-run inflation target for many CBs is 2 percent. The empirical literature usually finds that the technical output capital ratio (v) is about 0.3. Some empirical studies for the U.S. economy suggest that the CICU is about 82 percent (Corrado and Mattey 1997). The value assigned to ϕ stems from results reported in McElhattan (1985), who finds that for each percentage point that capacity utilization exceeds 82 percent, inflation accelerates by about 0.15 percentage points. The resulting value for r^* is less than half a percentage point above the average value for the real federal funds rate over the 1960–1998 period in the United States: 2.55 percent (Reifschneider and Williams 2000). Finally, the values of the parameters in the saving rate and investment function were chosen so as to render the model stable and, for the sake of convenience, we set $\bar{f}$ equal to its steady growth value.

FIGURES AND TABLES

Table 1: Values of the parameters, initial conditions, operators, and partial derivatives

$f_u = 0.05$	$s_{\hat{y}} = 1.5$	$\alpha = 0.5$	$\bar{u}_0 = 0.8$	$\partial r^* / \partial g_n = 1.0\widehat{7}$
$u^d = 0.8$	$\psi = 0.015$	$\alpha^* = 0.0\hat{6}$	$\Delta_1 = -0.0412$	$\partial r^* / \partial \pi^d = 0.0\widehat{6}$
$\pi^d = 0.02$	$v = 0.3$	$\bar{f} = 0.125$	$\Delta_2 = -0.00205$	$\partial r^* / \partial \bar{s} = -0.6$
$\bar{s} = 0.1$	$\phi = 0.15$	$r^* = 0.029\widehat{6}$	$\Delta_3 = 0.052$	$\partial r^* / \partial s_r = -0.019\widehat{7}$
$s_\pi = -0.1$	$g_n = 0.03$	$\pi_0 = 0.03$	$\Delta_4 = 0.00078$	$\Pi_0 = 0.0\widehat{6}$
$s_r = 1.5$	$\mu = 0.01$	$u_0 = 0.8$	$\partial \Delta_4 / \partial \alpha = 0.0018$	$\Pi_1 = -0.\widehat{6}$

Fig. 2: Inflation rate

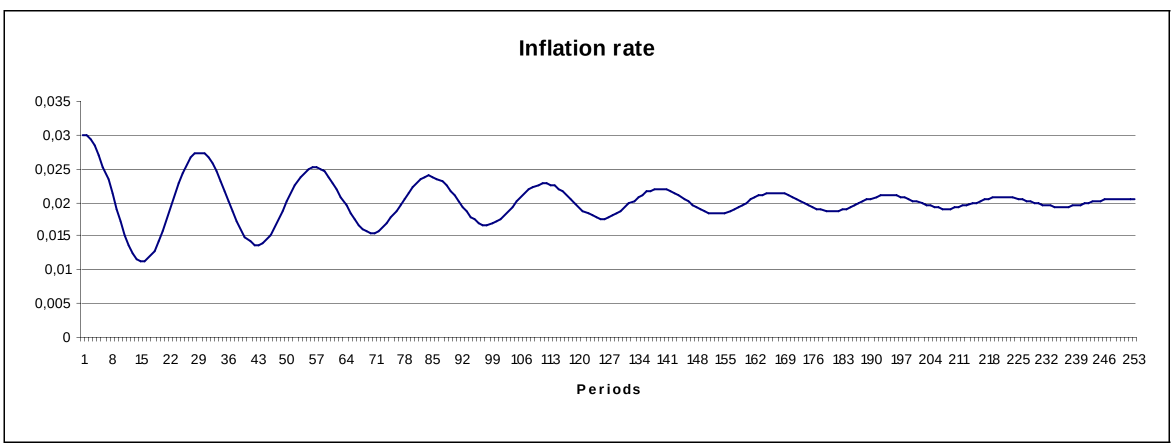


Fig. 3: Capacity utilization

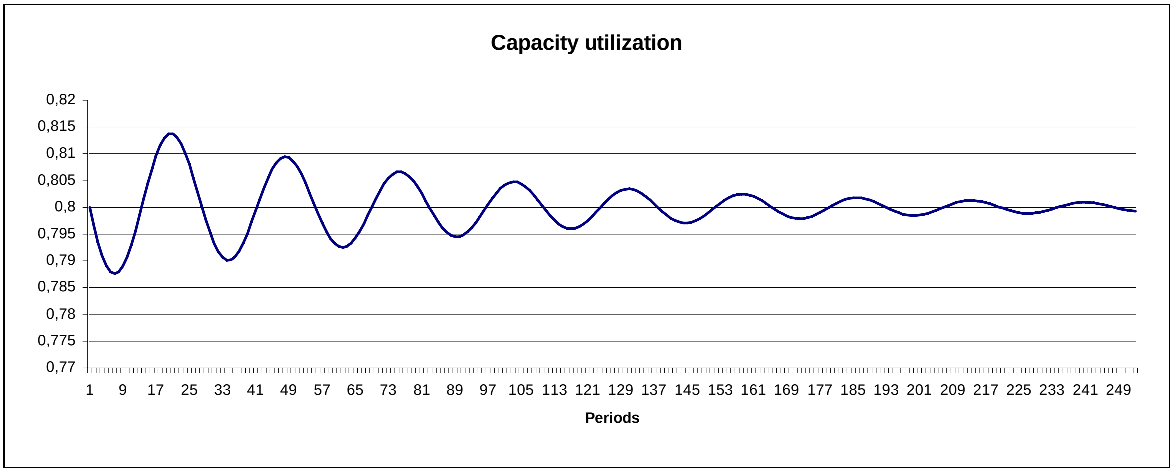


Fig. 4: Constant inflation capacity utilization (CICU)

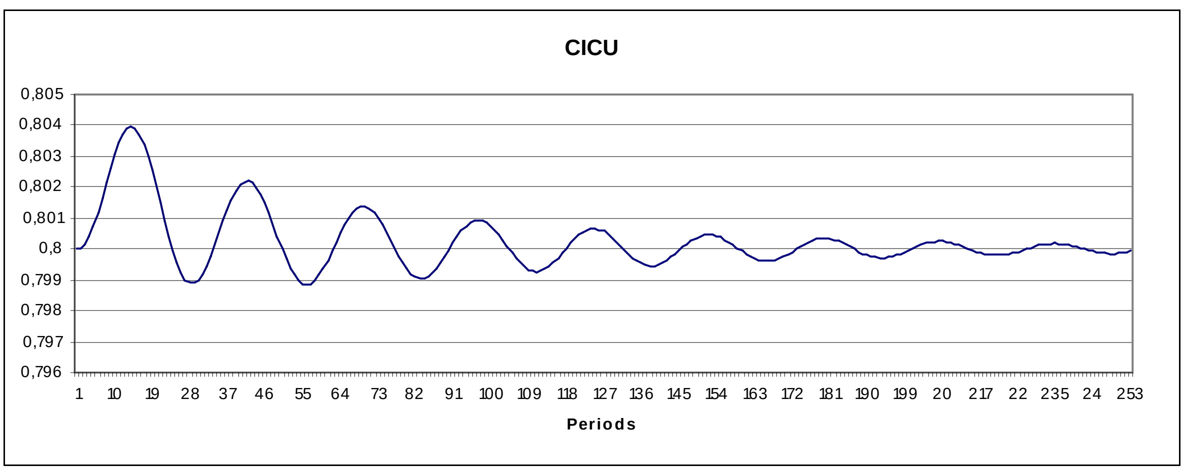
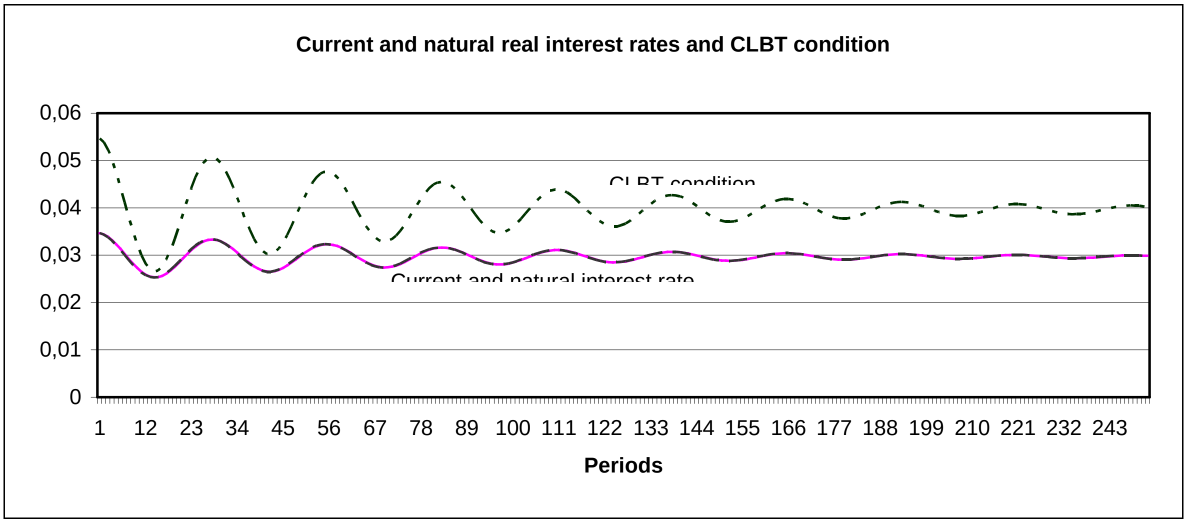


Fig. 5: Current and natural real interest rate and CLBT condition ($r^n + \pi - \mu$)



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