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When to Ease Off the Brakes (and Hopefully Prevent Recessions)

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ABSTRACT

Increases in the federal funds rate aimed at stabilizing the economy have inevitably been followed by recessions. Recently, peaks in the federal funds rate have occurred 6–16 months before the start of recessions; reductions in interest rates apparently occurred too late to prevent those recessions. Potential leading indicators include measures of labor productivity, labor utilization, and demand, all of which influence stock market conditions, the return to capital, and changes in the federal funds rate, among many others. We investigate the dynamics of the spread between the 10-year Treasury rate and the federal funds rate in order to better understand "when to ease off the (federal funds) brakes."

KEYWORDS: Federal Funds Rate; Yield Curve; Monetary Policy; Nonlinear Dynamics; Takens' Embedding

JEL CLASSIFICATIONS: C40; C60; E17; E42; E52

1. INTRODUCTION

Many authors have found the yield curve, specifically the spread between the interest rates on the 10-year Treasury note and the 3-month Treasury bill, to be a useful predictor of recessions (Estrella and Hardouvelis 1991; Estrella and Mishkin 1996, 1998; Estrella and Trubin 2006; Liu and Moench 2014). In particular, Liu and Moench (2014) explored combinations of this spread—the spread lagged six months (that is, the spread six months previous to the date under consideration)—and one of a variety of other economic variables as predictors. This work raises several questions: Are there other useful measures of the yield curve? Is the six-month lag optimal? Can one reconstruct a business cycle from yield curve data? Can the prediction algorithm inform control of the (monetary side of) the economy to "prevent" recessions?

Here we address these issues using tools from mathematical dynamics, in particular the Takens' (1981) embedding theorem (see also Broomhead and King [1986] and Sauer, Yorke, and Casdagli [1991]) and nonlinear smoothing (locally weighted scatterplot smoothing, or lowess) (Cleveland 1979; Cleveland and Devlin 1988) as implemented by Warnes et al. (2016) in the computer language R (R Core Team 2017) for separation of time scales (Kreiss 1979; Zhang, Mykland, and Aït-Sahalia 2005; Brackbill and Cohen 2014). The use of the spread as a predictor of recessions suggests that the spread (or its components) is a useful *observable* in the language of control theory (c.f., Hermann and Krener 1977). While control theory has antecedents from the 19th century (Maxwell 1868), contemporary antecedents begin with Kalman (1960), and Kalman, Falb, and Arbib (1969), who developed an efficient linear updating procedure with broad application.

Estrella and Trubin (2006) describe how monetary policy and investor expectations affect the slope of the yield curve. They explain,

> a tightening of monetary policy usually means a rise in short-term interest rates, typically intended to lead to a reduction in inflationary pressures. … Whereas short-term interest rates are relatively high as a result of the tightening, long-term rates tend to reflect longer term expectations and rise by less than short-term rates. The monetary tightening both slows down the economy and flattens (or even inverts) the yield curve. Changes in investor expectations can also change the slope of the yield curve [defined by these authors as the difference between the 10-year rate and the 3-month rate]. Consider that expectations of future short-term interest rates are related to future real demand for credit and to future inflation.

As a consequence, Estrella and Hardouvelis (1991) observed that the "slope of the yield curve is a good predictor 4 quarters ahead of a recession."

More recently, Bauer and Mertens (2018) defined the yield curve as the difference between the 10-year and the 1-year rates. As quoted in Phillips (2018), this yield curve "correctly signaled all nine recessions since 1955 and had only one false positive, in the mid-1960s, when an inversion was followed by an economic slowdown but not an official recession."

The rest of this paper is organized as follows: section 2 proposes a reconceived spread and its predictive ability; section 3 undertakes nonlinear dynamics of the spread via Takens' embedding after separation of time scales; section 4 discusses why dimension ~2 is reasonable; and section 5 concludes with a discussion of limitations and implications for future research.

2. SPREAD VERSUS FEDERAL FUNDS RATE

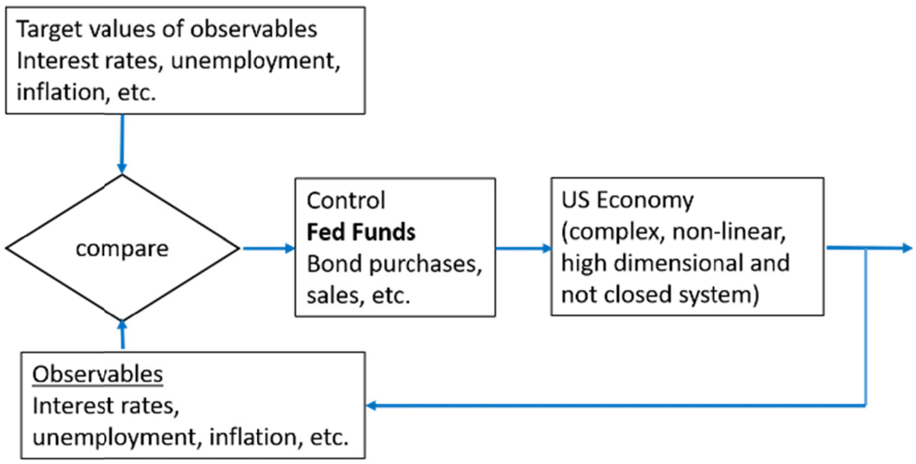
Motivated by control theory, we propose and consider here a modified definition of the spread, namely the 10-year rate minus the effective federal funds rate. In terms of the Fed monthly time series:

$$\begin{aligned} \text{Spread} &= \text{10-year rate} - \text{effective federal funds rate} \\ &= \text{GS10} - \text{FEDFUNDS} \qquad (1) \end{aligned}$$

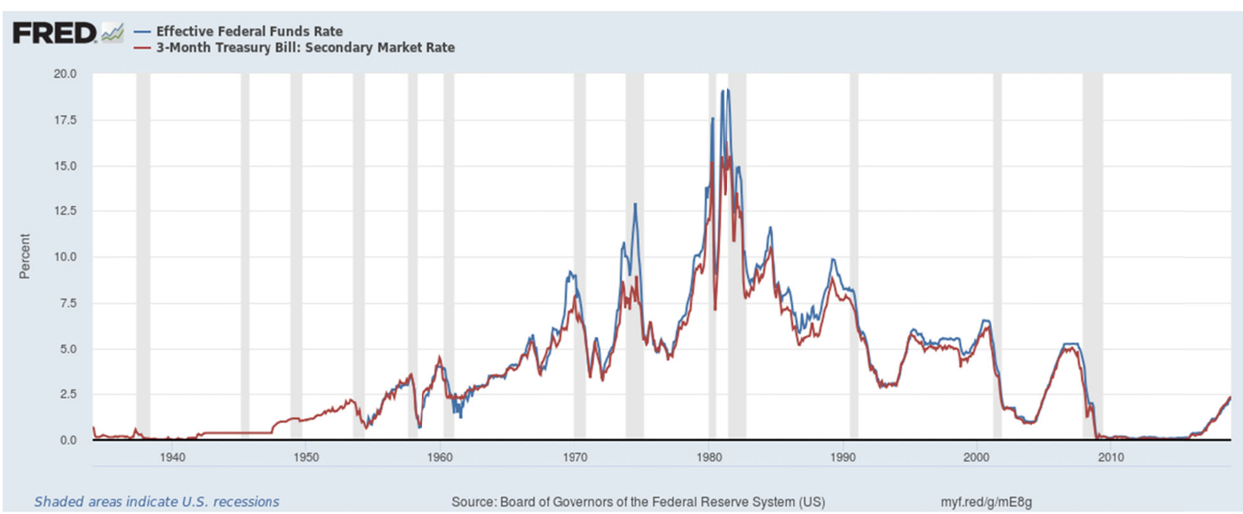
Unless otherwise noted, the terms "yield curve" and "spread" shall refer to this definition. Note that this spread was one of the variables considered but rejected by Liu and Moench (2014). Whereas the Fed only controls the 3-month rate indirectly, it does essentially directly control the effective federal funds rate, which may thus be regarded as a *control variable* in the language of control theory (figure 1A). In addition, the Fed exerts at least indirect control over the 10-year rate through control of the balance sheet and, in particular, the mix of government refinancing. Finally, there is a close relationship between the federal funds rate (FEDFUNDS) and the 3-month rate (TB3MS), as shown in figure 1, so that this spread is closely related to the more traditional spread.

Figure 1. The Role of the Effective Federal Funds Rate in Defining the Spread

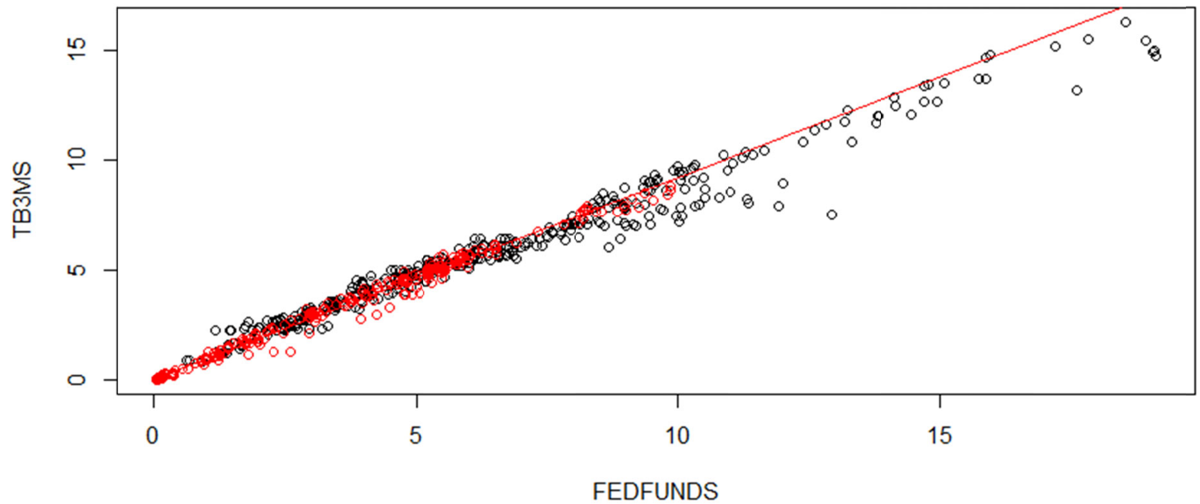
A. A Control Theory View



B. Comparison of Effective Federal Funds Rate and 3-month Treasury Bill Rate



C. Correlation Between Federal Funds Rate and 3-month Treasury Bill Rate



Notes: (B) Recessions shaded in gray.
(C) TB3MS versus FEDFUNDS, 1955-2018. Red indicates last 30 years, namely 1989-2018. Regression line for last 30 years using lm function in R:

$$\text{FEDFUNDS} = 0.918 \times \text{TB3MS} + 0.022,\ R^2 = 0.991,\ p < 2.2\times10^{-16}.$$

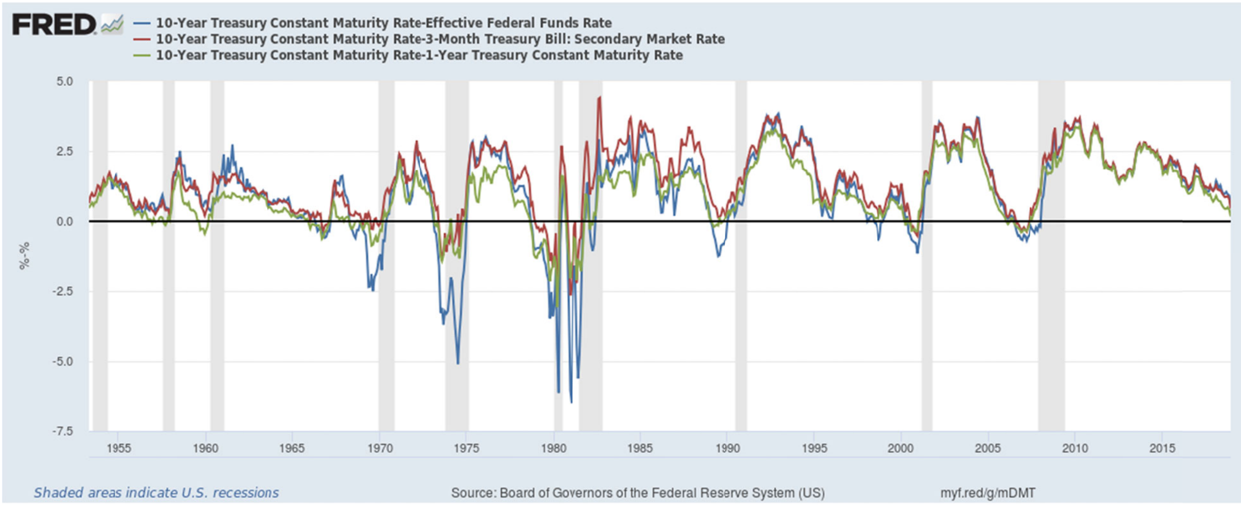
For the full series,

$$\text{FEDFUNDS} = 0.850 \times \text{TB3MS} + 0.350,\ R^2 = 0.979,\ p < 2.2\times10^{-16}.$$

Sources: (B) Reprinted with permission of Federal Reserve Bank of St. Louis. FRED Graphs (2019)
(C) Data from FRED, retrieved 1/13/2019.

Moreover, the spread between the 10-year Treasury constant maturity rate (GS10) and the effective federal funds rate (FEDFUNDS) defined in equation (1) is as effective as both the usual definition of the spread (GS10 – 3-month Treasury bill: secondary market rate [TB3MS]) and the spread between GS10 and the 1-year Treasury constant maturity rate (GS1) in predicting recessions in the last 50 years, and in fact gave a sharper signal of the dot-com bubble recession in the early 2000s (see figure 2 and table 1, below).

Figure 2. Time Series of Three Versions of the Spread



Source: Monthly data from FRED, retrieved 1/9/2019. Reprinted with permission of the Federal Reserve Bank of St. Louis.
Notes: 10-year Treasury constant maturity rate (GS10) – effective federal funds rate (FEDFUNDS) (blue); GS10 – 3-month Treasury bill: secondary market rate (TB3MS) (red); GS10 – 1-year Treasury constant maturity rate (GS1) (green)

Table 1. Sensitivity of Three Versions of the Spread

Spread	1955–67	1968–present
10-year Treasury constant maturity rate (GS10) – effective federal funds rate (FEDFUNDS)	Missed both recessions in the 1950s	100 percent
GS10 – 3-month Treasury bill: secondary market rate (TB3MS)	Missed both recessions in the 1950s	Missed recession in 1990s, slow to signal dot-com recession in 2001
GS10 – 1-year Treasury constant maturity rate (GS1)	100 percent	100 percent, but slow to signal dot-com recession in 2001

We next study the temporal evolution of the spread GS10 – FEDFUNDS, as shown in equation (1), using techniques from nonlinear dynamics.

3. NONLINEAR DYNAMICS OF THE SPREAD VIA TAKENS' EMBEDDING AFTER SEPARATION OF TIME SCALES

Since the spread GS10 – FEDFUNDS, as defined in equation (1) appears to be a useful predictor of recessions (c.f., Estrella and Hardouvelis 1991; Estrella and Mishkin 1996, 1998; Estrella and Trubin 2006; Liu and Moench 2014; Bauer and Mertens 2018), one may consider it to be an observable in the sense of control theory.

This observation leads to asking whether one can reconstruct (at least a useful part of) the dynamics of the US economy from the dynamics of the yield curve, using the Takens' embedding theorem. In particular, we use Takens' embedding and lag plots to model the dynamics of the yield curve, the latter following the program corrDim in the package fractal in R (Constantine and Percival 2017). The lag in Takens' embedding is determined using the first zero crossing of the autocorrelation, and is notably longer than the six-month and twelve-month lags used in Liu and Moench (2014). Lag plots of smoothed data can uncover significant linear and nonlinear patterns in data (c.f., Takens 1981; Sauer, Yorke, and Casdagli 1991; Hastings et al. 1996; Sauer 2006), while the Takens' embedding theorem shows that lagged data of one signal suffices to produce an embedding of *n*-dimensional dynamics (Takens 1981; Grassberger and Procaccia 1983; Sauer, Yorke, and Casdagli 1991; Sauer 2006).

3.1 Overview of Mathematical Concepts

Dynamical systems

The term "dynamical systems" refers to describing the temporal evolution of complex (dynamical) systems, usually by employing differential equations or difference equations. The state of a dynamical system at any given time can be represented by a vector in an appropriate state space. Its temporal evolution is thus represented by a trajectory in a state space; thus the state at any time is given by a tuple of real numbers (a vector) in an appropriate state space. Casdagli (1991) reviews the use of dynamical systems in modeling input-output systems.

Attractor

Roughly speaking, an attractor for a dynamical system is a closed subset A of its state space toward which trajectories in a neighborhood of the attractor tend to evolve, even if slightly perturbed (Milnor 2006). For example, the state space of a physical system may consist of positions and momenta of its components. The state space of an economic system may consist of variables such as the inflation rate, unemployment rate, gross domestic product, interest rates, etc. The business cycle can be considered as a trajectory in such an economic state space.

Thus, the first challenge in describing and understanding the dynamics of a complex system is to:

(a) find and identify a suitable finite-dimensional state space (if one exists);
(b) find and characterize an attractor within that state space (if one exists); and finally,
(c) describe the temporal evolution of the system on the attractor.

Even a good approximation can be useful. In many cases, the Takens' embedding theorem (see also Broomhead and King [1986] and Sauer, Yorke, and Casdagli [1991]) is a useful starting point. The Takens' embedding theorem states that one can reconstruct an *n*-dimensional attractor for a dynamical system through plotting sequences of lagged observations $\left(y(t), y(t-\tau), y(t-2\tau), \ldots, y(t-2n\tau)\right)$ of one signal (coordinate, observable) from that attractor (Sauer [2006]; see also supplemental material to Sugihara et al. [2012]) for an appropriate time lag τ. Even without knowing *n*, these lag plots can convey useful information about the attractor.

Correlation Dimension

The complexity of an attractor can be characterized by its correlation dimension. The correlation dimension is a natural generalization of familiar formulas for area and volume. The area of simple two-dimensional (2D) geometric objects is represented by the product of two linear dimensions, for example, $A = \text{length} \times \text{width}$ for a rectangle and $A = \pi r^2$ for a (solid) circle. The volume of simple three-dimensional (3D) geometric objects is represented by the product of three linear dimensions, for example, $V = \text{length} \times \text{width} \times \text{height}$ for a rectangular parallelepiped and $V = 4/3\ \pi r^3$ for a (solid) sphere. Finally, since the circumference of (the boundary of) a circle is $C = 2\pi r$, the boundary is one dimensional. The natural dimension of attractors reconstructed by the Takens' embedding theorem, which are simply sets of points, is an exponent D for which the number of points within a distance r of a typical point scales is:

$$D \sim const \times r^D \tag{2}$$

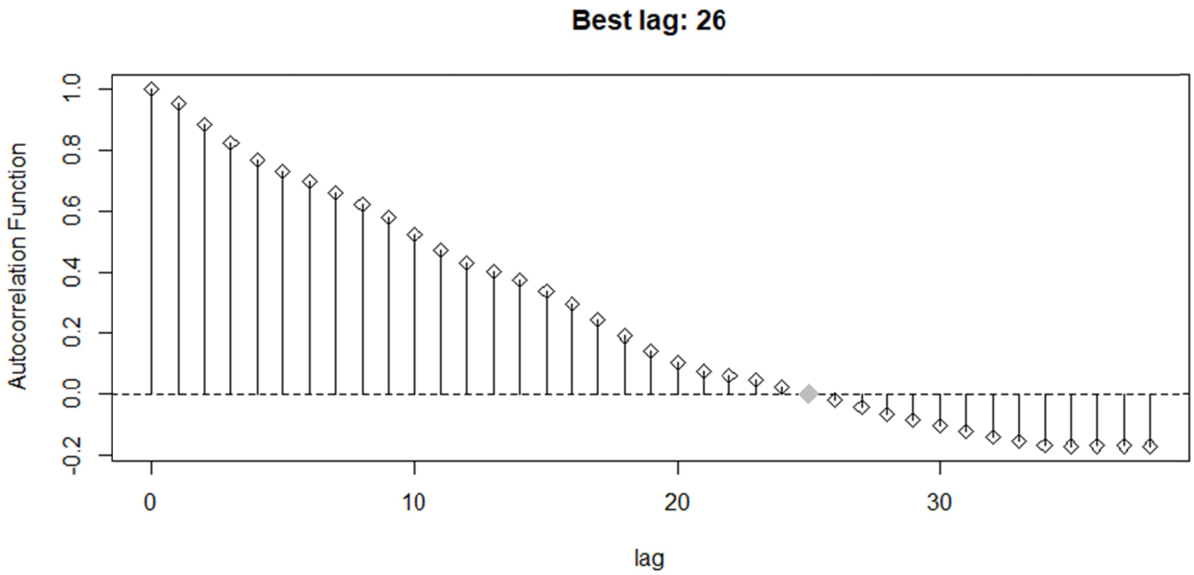
Note that the correlation dimension need not be an integer since it is an example of a fractal dimension. The use of the correlation dimension to characterize an attractor is known as the Grassberger-Procaccia (1983) algorithm (c.f., Grassberger [2007] for a review).

Application of these methods to time series data is limited by the length of the time series. Eckmann and Ruelle (1992) show "that values of the correlation dimension estimated over a decade from the Grassberger-Procaccia algorithm cannot exceed the value 2 $\log_{10}$N if N is the number of points in the time series."

3.2 The Time Lag

In general, there is no optimal time lag τ for attractor reconstruction via the Takens' embedding theorem. We have used one of the standard methods, namely, the first zero of the autocorrelation function: the correlation of $y(t)$ and $y(t-\tau)$. For example, if $y(t) = \cos(2\pi t/T)$, a simple periodic function with period T, this method finds the time lag $\tau = T/4$. Note that $y(t-\tau) = y(t-T/4) = \sin(2\pi t/T)$. We found $\tau =$ 26 months for the time series of monthly spread data from 1955 to 2018 (GS10 – FEDFUNDS, 768 points), using the "timelag" program in the fractal package in R (Constantine and Percival 2017).

Figure 3. Autocorrelation of the Spread GS10 – FEDFUNDS



3.3. Correlation Dimension of the Spread

Using the program corrDim in the package fractal in R (Constantine and Percival 2017) to explore the dynamics, we present the output in table 2.

Table 2. Dynamics of the Spread: Output from the Program corrDim

Correlation dimension for GS10 – FEDFUNDS (586 points, time lag 26, L-inf metric)

Embedding dimension	1	2	3	4	5	6	7	8
Correlation dimension (invariant estimate)	1.101	1.939	2.632	2.641	3.585	3.487	3.504	3.521

The correlation dimension approached 3.5 in embedding dimension 8, with a lag of 26 months (using the first zero crossing of the autocorrelation function, shown in figure 3, below), yielding a tangled lag plot in two dimensions (figure 5A, below). Given the limited amount of data, we elected to detrend and somewhat smooth the data, with an aim to determining dynamics on a reasonable time scale by removing shorter-term "noise" and any longer-term trend. We therefore next smoothed the spread time series with lowess (Cleveland 1979; Cleveland and Devlin 1988) as implemented in R (Warnes et al. 2016). Frequent sharp peaks and falls of economic time series may be attributed to noise. The use of lowess performs a nonlinear separation of time scales so that Takens' embedding focuses on an empirically appropriate timescale, yielding a ~2D attractor in 3D space.

3.4 Smoothed Data

A graph of the time series of the spread (figure 2) suggests a time scale of 7–10 years for a full "cycle" and thus the 26-month lag above approximates a quarter of the cycle. For reference, consider a sine wave of period T, represented by the formula: $sin\big((2\pi/T)t\big)$. The corresponding cosine wave $cos\big((2\pi/T)t\big)$ is uncorrelated with the sine wave (the integral over any whole number of periods $\int sin\big((2\pi/T)t\big)cos\big((2\pi/T)t\big)dt = 0$) and lags the sine wave by a quarter period, namely $T/4$: $cos\big((2\pi/T)t\big) = sin\big((2\pi/T)t - \pi/2\big) = sin\big((2\pi/T)(t - T/4)\big)$. In order to better focus on dynamics over this time scale, we introduce the use of lowess smoothing, a nonlinear smoothing technique, to effect a separation of time scales.

In addition to lowess, there are several standard algorithms to highlight essential features that are hidden by the noise, e.g., moving averages and Gaussian filters (c.f., Hyndman 2011) and wavelets (c.f., Yi, Li, and Zhao 2012). We used lowess rather than moving averages for detrending because: (1) the trend (dynamic on scales > 7–10 years) may be nonlinear and subtracting a moving average would assume a linear trend; and (2) smoothing the "very fast" dynamics on scales < 26 months might be accomplished by lowess or more local smoothing techniques (e.g., Gaussian filters). We chose lowess because it is also used to remove the trend.

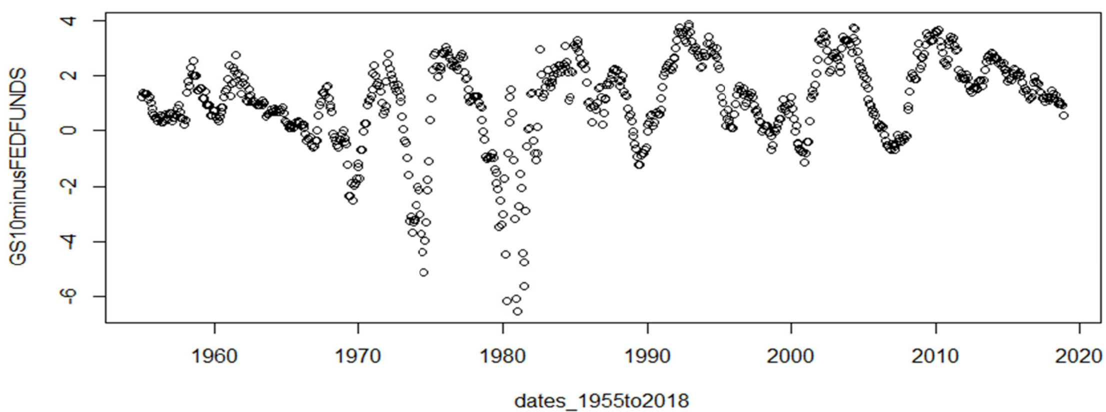
In sum, we study smoothed, detrended dynamics as follows:

a. Detrending, where the trend is computed using a lowess smoother (using default parameters in the gplot package in R [Warnes et al. 2016]);
b. Smoothing (lowess, with lowess parameter f=.05);
c. Computing the correlation dimension, using the program corrDim in the package fractal in R (Constantine and Percival 2017).

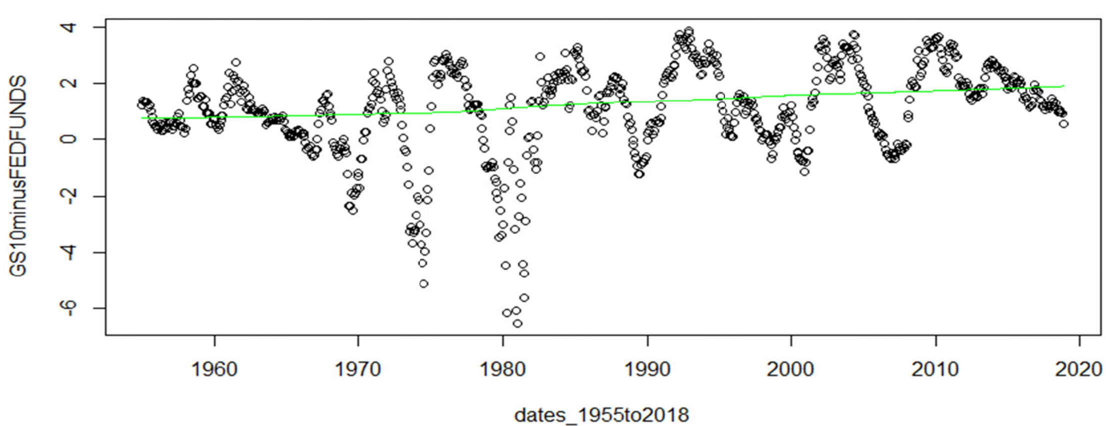
This process is summarized in figure 4 and the resulting dynamics are shown in figure 5B. Note the same 26-month lag, but now the correlation dimension stabilizes at ~2 (table 3).

Figure 4. Detrending and Smoothing the Time Series of the Spread GS10 – FEDFUNDS (768 points)

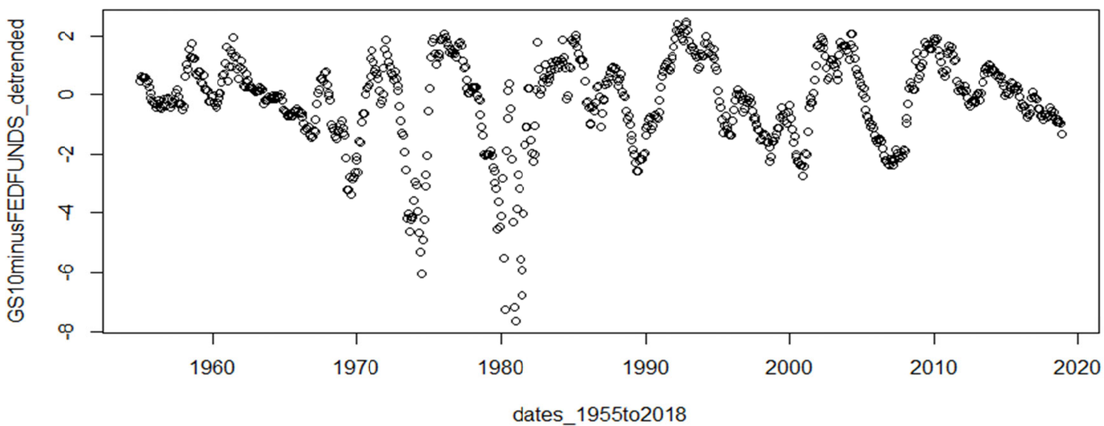
A. The Original Time Series of the Spread GS10 – FEDFUNDS



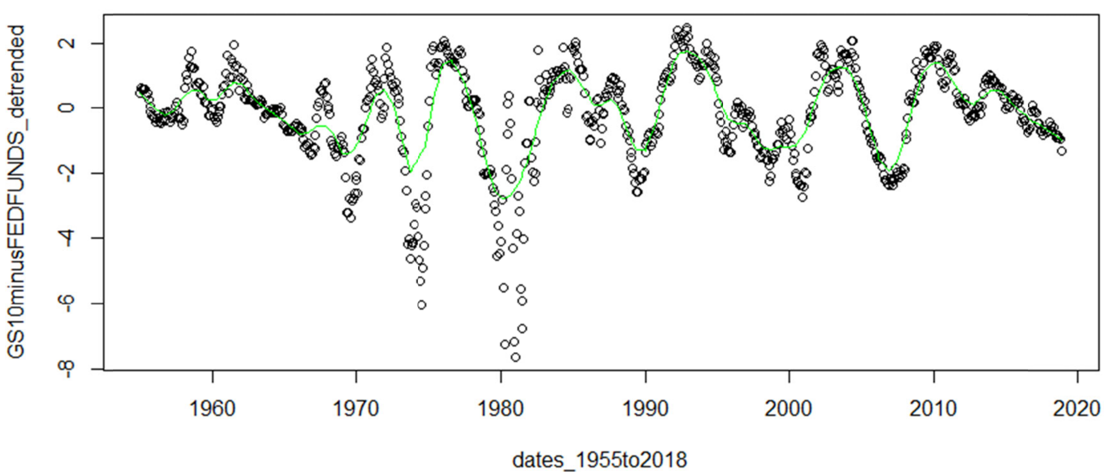
B. The Long-term Trend (green line) Determined by lowess



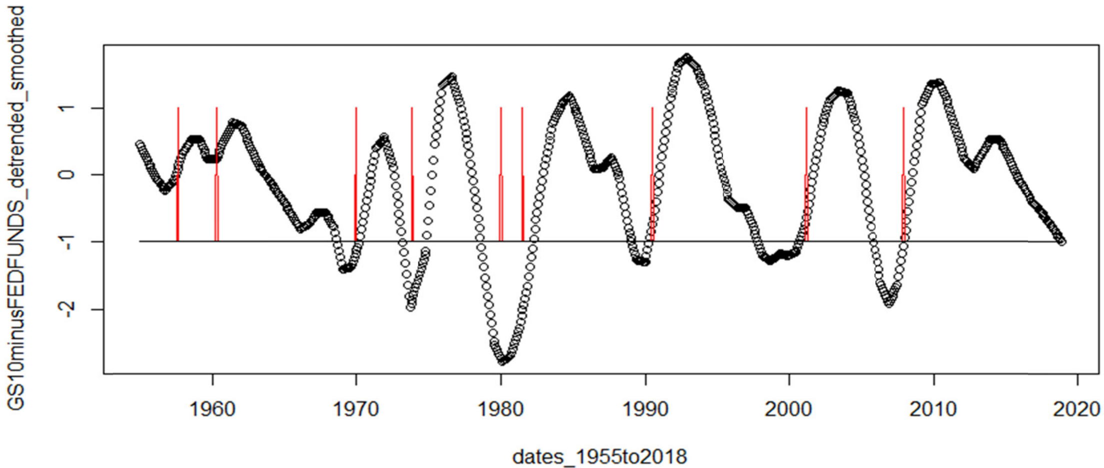
C. The Detrended Spread



D. The Smoothed, Detrended Spread (a second application of lowess), Shown as a Green Curve Superimposed on the Detrended Spread



E. The Smoothed, Detrended Spectrum as a Predictor of Recessions



Notes: (A) The time series of the spread GS10 – FEDFUNDS (768 points); (B) The trend (green line) found using lowess; (C) The detrended spread (note change in vertical scale); (D) The smoothed, detrended spread (green line), found using lowess with $f = 0.05$; (E) decrease of the smoothed detrended spread below -1 percent forecasts recessions starting in 1968 (starting dates of NBER recessions are shown in red, data from FRED using the midpoint method)
Source: https://fred.stlouisfed.org/series/USRECM, retrieved 1/9/2019.

Table 3. Dynamics of the Smoothed, Detrended Spread: Output from the Program corrDim

Correlation dimension for GS10minusFEDFUNDS_detrended (586 points, time lag 26, L-inf metric)

Embedding dimension	1	2	3	4	5	6	7	8
Correlation dimension (invariant estimate)	0.976	1.607	1.565	1.986	2.057	2.051	2.256	2.503

Figure 5A. Lag Plots of the Spread GS10 − FEDFUNDS

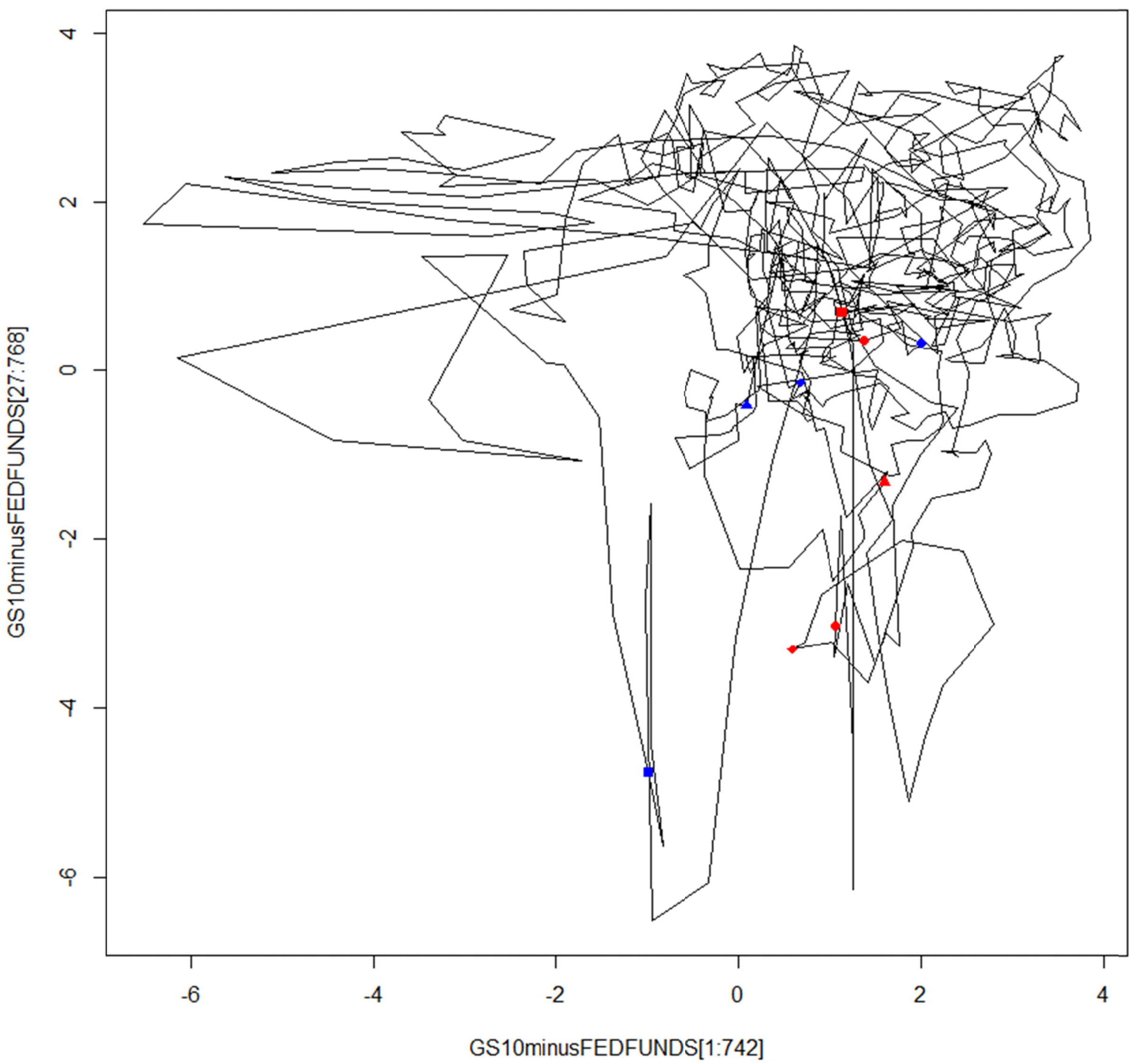
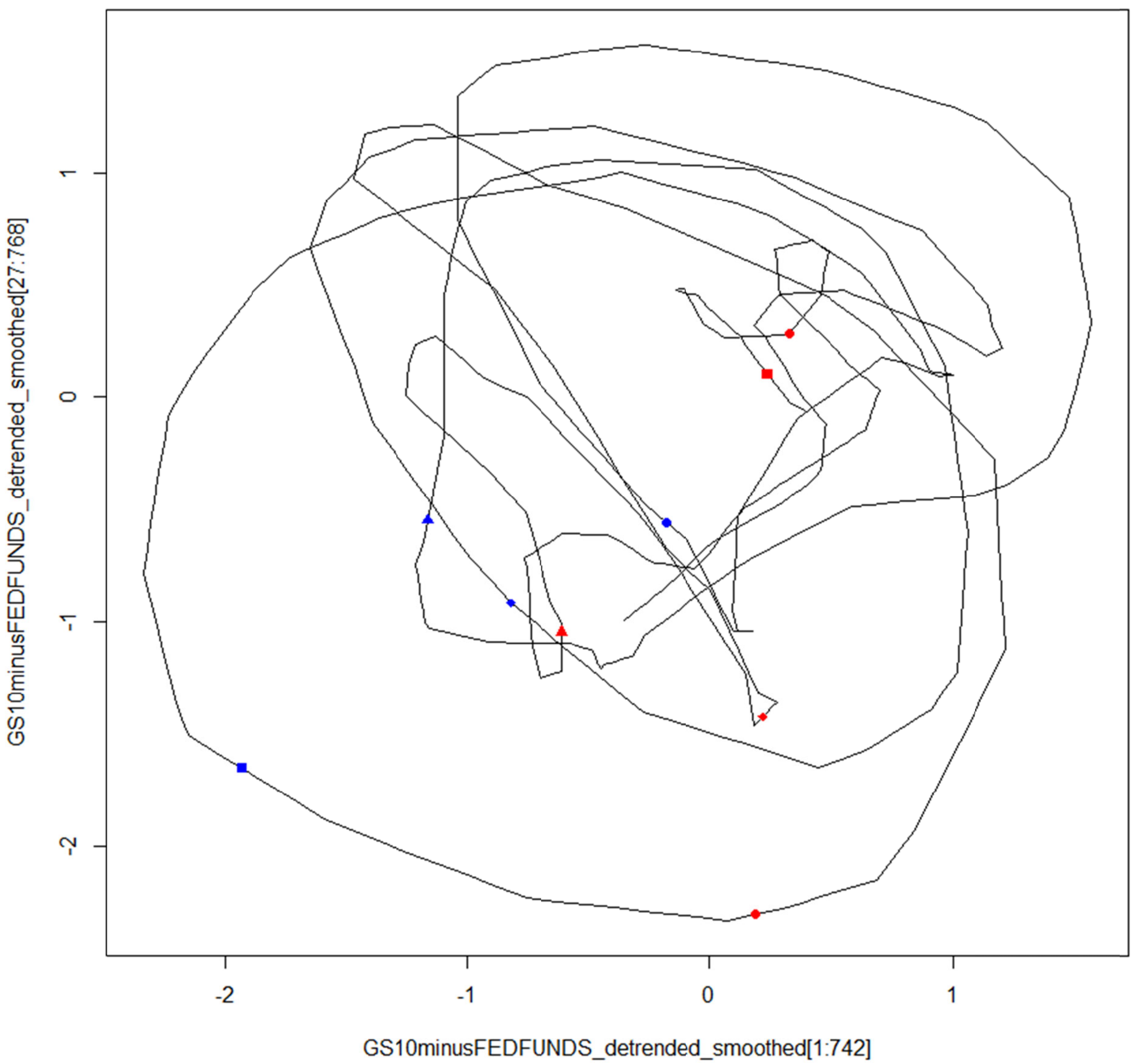


Figure 5B. The Smoothed, Detrended Spread



Notes: Lag plots of the spread GS10 – FEDFUNDS (figure 5A) and the smoothed, detrended spread (figure 5B), both projected into two dimensions. The correlation dimensions of the corresponding attractors are ~3.5 (the spread) and ~2 (the smoothed, detrended spread). The general direction of motion along the attractor is counterclockwise. Starting dates of recessions are shown by colored markers as in table 4.
Source: NBER, FRED data series USRECM, retrieved 1/9/2019.

Table 4. Legend for Figures 5A and 5B: Starting Dates of Recessions

Starting months of recessions (NBER, source: FRED data series USRECM)

1957-08-01	1981-07-01
1960-04-01	1990-07-01
1969-12-01	2001-03-01
1973-11-01	2007-12-01
1980-01-01	

Hence, it appears we can capture cyclic dynamics with the spread, perhaps indicating the credit/liquidity cycle in the macroeconomic system (Kindelberger 2006; Minsky 1986).

4. WHY DIMENSION ~ 2 IS REASONABLE

We now consider whether and why the dimension is about 2, looking at both Liu and Moench's (2014) predictive models, and two dynamical models: the Goodwin (1967) model and the more recent Bar-Yam et al. (2017) model.

Liu and Moench (2014) used the nonlinear probit model (c.f., Spermann 2008) to develop predictors of future recessions and found that "usual" spread (10 year minus 3 months, that is GS10 − TB3MS) and six-month lagged spread were optimal for predicting recessions within time horizons of 6–24 months. The probit model aims to forecast the probability that a recession will occur under given conditions. Moreover, Liu and Moench used a receiver operating characteristic (ROC) curve to evaluate the predictive ability of a given model; the area under the ROC curve describes the tradeoff between the sensitivity and specificity as the criterion (probability level) in predicting a recession in a probit model. They observe that additional predictors—beyond spread (10 year minus 3 months) and 6-month lagged spread—added very little to the ROC curve, suggesting that the relevant dynamics could be largely captured in dimension ~ 2.

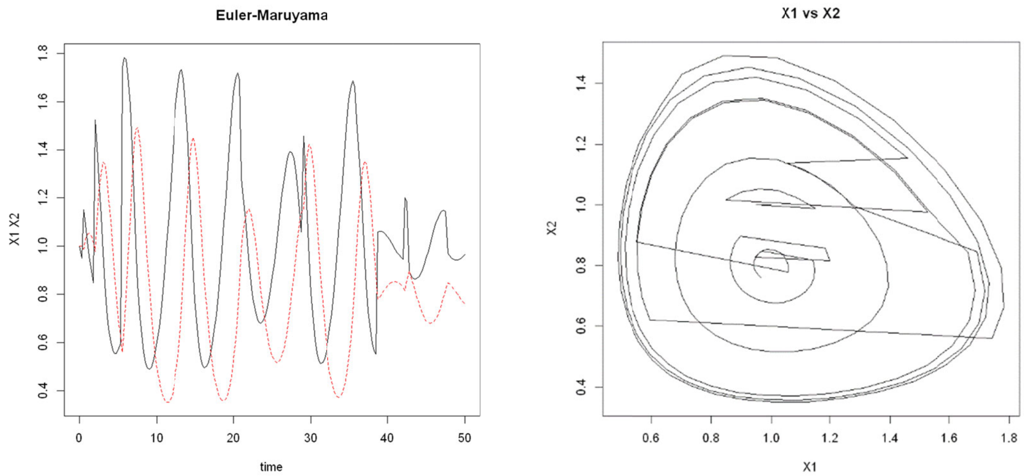
The Goodwin model (a two-variable model) combines Lotka-Volterra predator-prey dynamics[1] with an exponential growth term. Lotka-Volterra dynamics yield neutrally stable cycles, so Goodwin-Lotka-Volterra dynamics are not structurally stable and may thus exhibit complex responses to noise (Velupillai 1979; Pohjola 1981; Flaschel 1984; Boldrin and Woodford 1990; Desai et al. 2006; Veneziani and Mohun 2006). A neutrally stable cycle is a one-dimensional object (like a circle). A cycle, with amplitude varying randomly within bounds, will generate trajectories in an annulus of dimension = 2. Lotka-Volterra dynamics density-dependent growth plus noise is readily seen to generate noise-driven limit cycles of dimension ~ 2, as seen below. Equations for the noise-driven cycle with jump processes can be given as:

$$\begin{aligned} &\text{Capital, prey:} \quad && \Delta x_1 = \theta_1 x_1 (1 - x_1) - \gamma_1 x_1 x_2 + \sigma_1 x_1 x_2 \Delta w + jump \\ &\text{Labor, predator:} \quad && \Delta x_2 = \gamma_2 x_1 x_2 - \theta_2 x_2 + \sigma_2 x_1 x_2 \Delta w \end{aligned} \tag{3}$$

Figure 6 shows the typical results of such a simulation, integrated with the Euler-Maruyama algorithm, a standard algorithm for numerical integration of stochastic differential equations (Higham 2001).

[1] Lotka (1924); see also the reviews by Hoppensteadt (2006) and Baigent (2010). Barbosa, Filho, and Taylor (2006) combine similar dynamic combined with growth. In the terminology of Lotka-Volterra models, the workers' share is the "predator" and the capital share the "prey" (Veneziani and Mohun 2006; Huu,Nguyen, and Costa-Lima 2014). See also Harvie (2000) and Stockhammer and Michell (2016) for Goodwin dynamics.

Figure 6. Lotka-Volterra Dynamics with Noise



Notes: Left: Time series. Prey denoted X1 (solid black line), predators X2 (dashed red line). Right: Dynamics in the X1-X2 state space. The correlation dimension stabilizes at ~2.

Here we can see both of the cyclic dynamics indicative of something like the credit/liquidity cycles, as well as jump processes, capturing excess volatility not accounted for in the baseline model. Such dynamics indicate an approximately 2D model may serve well to account for baseline macro-attractors.

Bar-Yam et al. (2017) model an investment and consumption cycle, which is roughly analogous to Goodwin dynamics, but more complex. As they state: “Thus the models describe economic and monetary dynamics distinguishing two primary monetary loops (1) consumption and wages, and (2) investment and returns (rents). Such a framework is manifest, for example, in the Goodwin (1967) model and also the Kalecki (1954) model, used to describe macroeconomic oscillations called ‘business cycles.’” We therefore decided to study the description of cycles of investment and consumption that led to the Bar-Yam et al. model.

Figure 7 shows investment and consumption time series data from FRED (top) and detrended investment and consumption data (bottom); that is, it shows ratios of investment and consumption to exponential trends in the respective data. Consumption grew at an annual rate of 0.253 percent after inflation and seasonal adjustment (R^2 = 0.913); investment grew at an average annual rate of 0.648 percent (R^2 = 0.597, reflecting larger fluctuations about the exponential trend line). As shown by Bar-Yam et al. (2017), investment shows quasicyclic behavior, peaking just before recessions. Takens' embedding following lowess smoothing and separation of time scales also yields dimension ~2 (figure 8).

Figure 7A. Investment and Consumption Time Series: Investment and Consumption, Raw Data

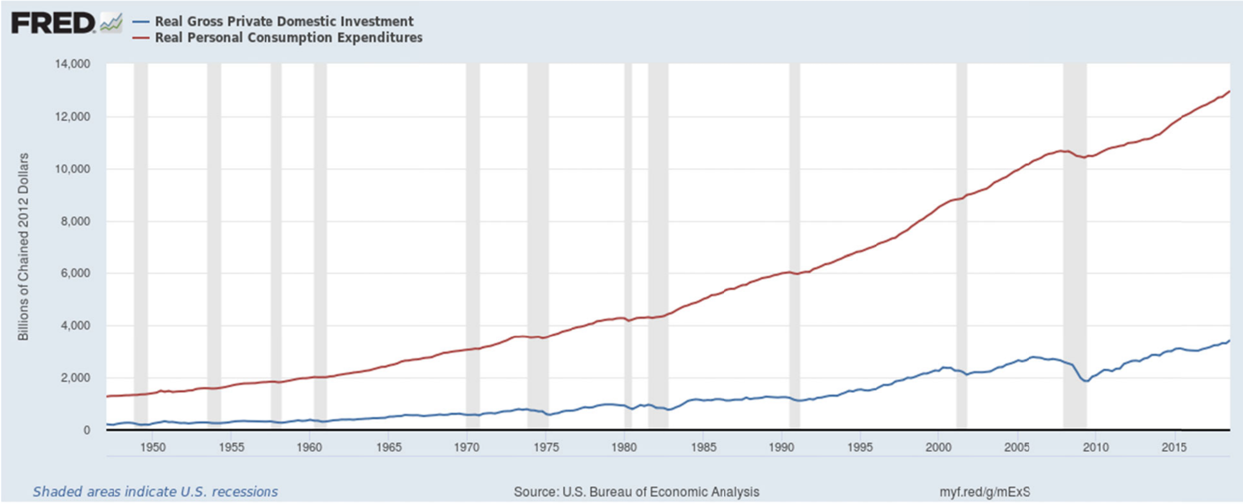
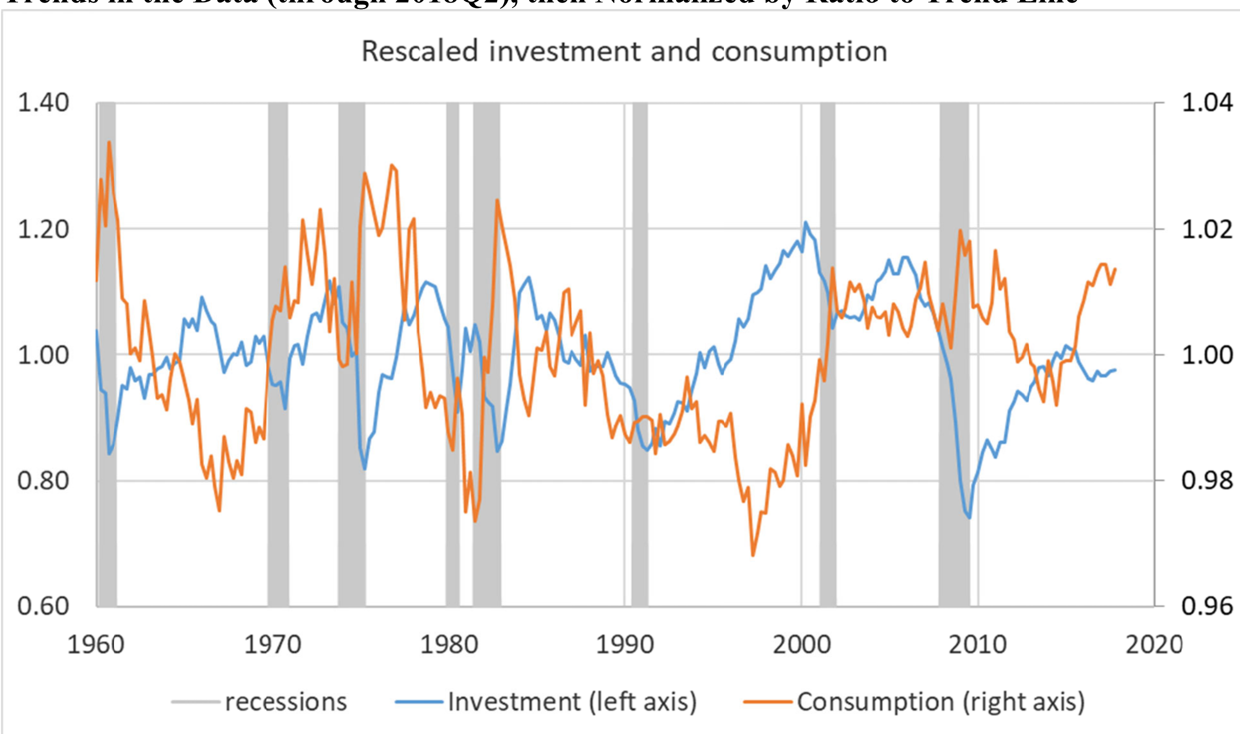
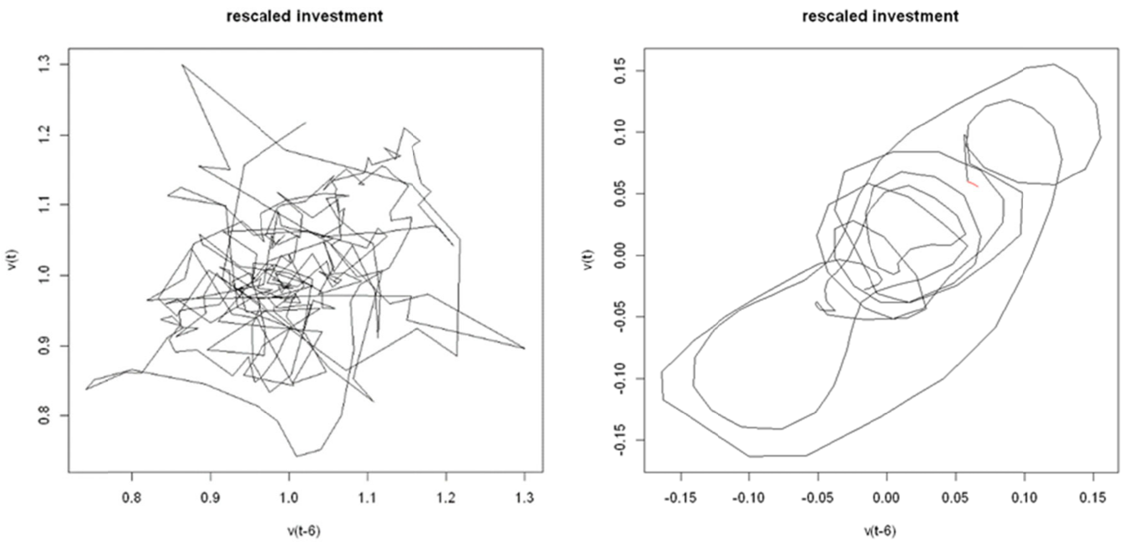


Figure 7B. Investment and Consumption Data Detrended by First Removing Exponential Trends in the Data (through 2018Q2), then Normalized by Ratio to Trend Line



Sources: U.S. Bureau of Economic Analysis, Real Gross Private Domestic Investment [GPDIC1], and U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures [PCECC96], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GPDIC1 and https://fred.stlouisfed.org/series/PCECC96, respectively, January 14, 2019.

Figure 8. Lag Plots of Rescaled Investment, lag = 6 (arbitrary units): No Further Processing (left) and after lowess Detrending and Smoothing (right)

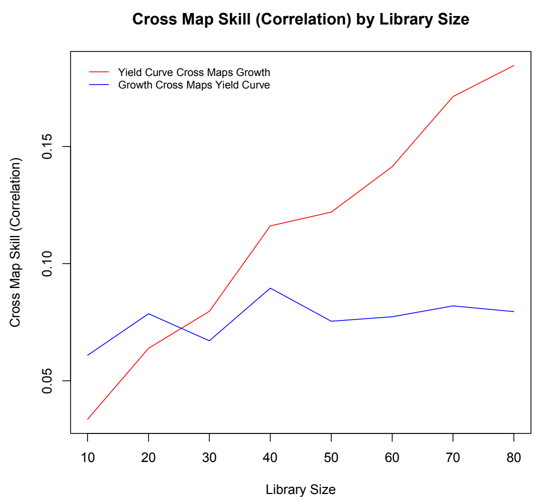


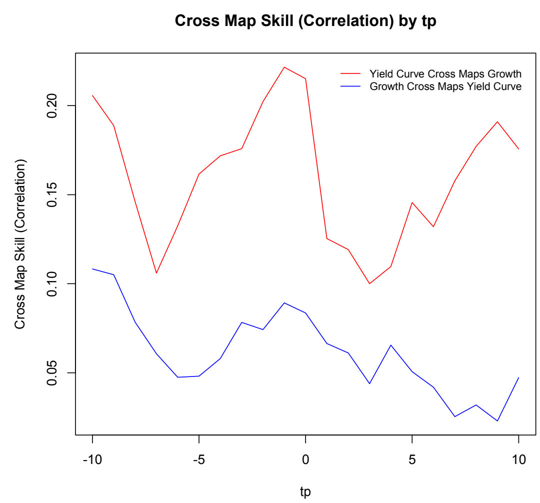
Notes: Red mark at right indicates start of dynamics (earliest points). Note complex dynamics before lowess processing, and smoother dynamics (reflecting dimension ~ 2) after processing.

In summary, the dimension analysis of the spread, following lowess separation of time scales, yields results (dimension ~ 2) consistent with those of typical simplified economic models. As considered, both yield measures of the monetary base relative to maturity class as well as macroeconomic aggregate flows of investment and consumption, and both seem to pick up such cyclical dynamics, suggestive of competitive business cycles in a financial economy.

Finally, we consider Sugihara causality (Sugihara et al. 2012) from the yield curve to GDP growth. Our abstract with a student-lead author (Wang, Young-Taft, and Hastings 2018) found evidence for Sugihara causality from the yield curve (GS10–FEDFUNDS) to GDP growth (note increasing correlation with increasing "library size" [number of points used in determining correlation] in figure 9, left), as well as evidence for a cycle in causality with peaks at time offsets of ±7–10 quarters in addition to the peak at lag 0 (see figure 9, right). Note there is an important *policy consequence*: the effect of changing the spread upon growth may not peak for 7–10 quarters.

Figure 9. Causality from the Yield Curve to GDP Growth (left-hand side) and the Effect of Time Offset (tp) on Correlation





Notes: In the left-hand graph, note increasing correlation with increasing "library size" (number of points used in determining correlation). In the right-hand graph, note peaks at ± 7-10.

We conclude with some discussion of our results, including limitations and potential for future work.

5. DISCUSSION

In this paper we studied the dynamics of the yield curve followed by suitable detrending and separation of time scales via lowess smoothing resulting in quasicyclic dynamics. One standard approach to Takens' (1981) embedding found the first zero of autocorrelation at a lag of 26 months, and a "macroeconomic attractor" of dimension ~ 2. The 26-month lag was shown to be consistent with the lag associated with monetary effects on the macroeconomy using Sugihara causality. We plan to extend the use of dynamic time series analysis in monetary and macroeconomic analysis, pointing the way toward future applications across variables and economic settings, including: (1) sectoral analysis; (2) analysis of yield curves with multiple maturities; (3) market microstructure analysis of Treasury issuance and limit order and order books; (4) inclusion of various yields of different financial instruments in a multivariate exercise; and (5) comparison between and across countries, undertaken perhaps under the null hypothesis of ~ 2 dimensional dynamics.

Limitations

Although we find this approach and results to be interesting, substantive, and demonstrative, potential attractor dynamics and topology via variable selection and modeling regimes still remain to be explored. One may also need to consider velocity (derivatives) of variables in variables themselves, as well as multidimensional embedding (Barnard, Aldrich, and Gerber 2001), data-driven equation-free approaches (Ye et al. 2015), and alternative filtering techniques—such as Takens-Kalman filtering (Hamilton, Berry, and Sauer 2016, 2017)—in order to generate fundamental, core predictive dynamical models.

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